OPTIMAL SHAPE OF LIFTING BODIES IN A FLOW WITH A PLANE SHOCK

V. V. Keldysh

Some possibilities of improving the lift-to-drag ratio of lifting bodies in a supersonic flow with a plane shock attached to the leading edges are analyzed.

The aerodynamic characteristics of such bodies, as well as of bodies in a flow with a conical/exponential shock, have been analyzed in numerous papers, e.g. [1-4], where attention is concentrated on the windward surface, which is a streamsurface in the flowfield behind a shock of a given configuration and generates a considerable fraction of the aerodynamic forces. Usually, the upper surface of the body is specified as a streamsurface of the undisturbed flow. Such bodies are called waveriders.

In a flow with a plane shock, waveriders have a ruled surface whose windward and upper parts are described by the equations \( y = x \tan \alpha + (\tan \gamma - \tan \alpha) y_0 \) and \( y = f(z) \tan \gamma \) respectively. Here, \( x_0 = f(z) \) is the equation of the leading edge projection onto the horizontal plane, whose \( x \)-axis coincides with the free-stream direction while the \( z \)-axis is perpendicular to the vertical symmetry plane of the body, \( \alpha \) is the windward surface angle of attack (angle through which the flow velocity vector rotates in the shock), and \( \gamma \) is the angle between the shock plane and the free-stream direction. The base cross section of the body is assumed to be perpendicular to the \( x \)-axis.

In inviscid gas flows the lift-to-drag ratio of the waverider is independent of the shape of its surface and is determined by the angle of attack \( \alpha \) and the nondimensional base pressure \( p_S^* \):

\[
K_w = \frac{p_S^* - 1}{p_S^* - p_b^*} \cot \alpha,
\]

where \( p_S^* \) is the nondimensional pressure behind the shock, \( p_S^* = p_i / p_b \) \( i = S, b \), and \( p_b^* \) is the static pressure upstream of the shock.

The friction drag depends on the body shape, which may have a considerable effect on the lift-to-drag ratio for a given flow regime (the free-stream numbers \( M \) and \( Re \) and the angle of attack \( \alpha \)), subject to certain geometrical conditions, for instance, a given value of the body volume coefficient. The optimal shape of a body having the maximum lift-to-drag ratio for a given flow regime with a plane shock and a given volume coefficient is chosen using the exact relations at the shock and an approximate formula for calculating the local friction coefficient

\[
\epsilon_f = \frac{C_i}{Re_i^{m}}, \quad i = S, \infty
\]

Here, \( C_i \) and \( Re_i \) are found from the local flow parameters at the boundary layer edge and the distance measured from the leading edge along the streamline.

Fully laminar \((m=0.5)\) and turbulent \((m=0.2)\) boundary layers on the body surface is considered. Both the interaction of the layer with the external flow and the real properties of air are ignored. For a laminar boundary layer coefficient \( C_i \) is calculated using the formula [5]

\[
C_i = \frac{(T_i + 110.4 \sqrt{h_i^*})}{T_i h_i^* + 110.4}, \quad h_i^* = 0.4605 + 0.5395 \frac{T_w}{T_i} + 0.0269 M_i^2,
\]

where \( T_i \) and \( T_w \) are the flow and body surface temperatures respectively. The latter is taken equal to 1100 K.

For a turbulent boundary layer [6]
\[ C_i = 0.0592 \left( \frac{T_i}{T_w} \right)^{0.27} \left( 1 + \frac{x \pm 1}{2} r M_i^2 \right)^{0.28}, \]

where \( x = 1.4 \) is the adiabatic exponent of the gas, and \( r = 0.845 \) is the total pressure restoration coefficient.

If the shock is plane, then the flowfields in the neighborhood of the lower and upper surfaces are uniform and the expressions for the lift-to-drag ratio become

\[ K = \left( \frac{F_s - 1}{F_s} \right) \left( \frac{F_s - p_s}{p_s} \right) \tan \alpha + \frac{1}{2n} \lambda M^2 C_{F_s} \left[ I(\gamma) + k(\gamma - \alpha) \right]. \]

\[ I(\xi) = \int_0^{z_0^*} \left[ 1 - f(z^*) \right]^{1-m} \left[ 1 + f(z^*) \sin \xi \right]^2 dz^*. \]

\[ k = \frac{q_s C_{F_s}}{q_s C_{F_{-i}}} \]

Here, \( L, S, \) and \( z_0^* \) are the length of the body projected onto the horizontal plane, its nondimensional area and half-span respectively, \( q_s \) is the dynamic pressure, \( C_{F_s} \) is the friction drag coefficient for a flat rectangular plate in a uniform flow, \( i = S \) and \( \alpha \) downstream and upstream of the plane shock respectively, and \( Re, \) is based on the body surface root chord.

For a given flow regime with a plane shock and a laminar/turbulent boundary layer on the body surface, the geometric parameters corresponding to the maximum lift-to-drag ratio are independent of the Reynolds number, because the lift-to-drag ratio extremum condition contains only the ratio of the plate friction drag down- and upstream of the shock, which is determined by the shock strength.

For the class of bodies with a two-parameter leading edge described by the formula \( x_0 = f(z^*) = (z^*/z_0)^n, \) \( n > 1, \) the expression for the lift-to-drag ratio becomes

\[ K = \left( \frac{F_s - 1}{F_s} \right) \left( \frac{F_s - p_s}{p_s} \right) \tan \alpha + \frac{1}{2n} \lambda M^2 C_{F_s} I, \]

\[ I = \int_0^1 \left[ 1 - t^n \right]^{-m} \left[ \sqrt{1 + Q(\gamma)} + k/\sqrt{1 + Q(\gamma - \alpha)} \right] dt, \]

\[ Q(\xi) = \frac{n \sin \xi}{z_0^* \cos \gamma} \]

For a given body volume coefficient

\[ \tau = \frac{U}{S^{3/2}} = \sqrt{\frac{n(1 + n)}{2z_0^*}} \frac{\tan \alpha}{2n + 1} \]

the optimal values of the parameters \( n \) and \( z_0^* \), corresponding to the maximum lift-to-drag ratio for a given flow regime, are found from the condition \( dK/dn = 0, \) which can be reduced to the equation

\[ I = n(1 + n) \left[ \frac{\partial I}{\partial n} + \frac{\tan^2 \alpha}{2 \tau^2 (1 + 2n)^3} \frac{\partial I}{\partial z_0^*} \right], \]

where \( \partial I/\partial n \) and \( \partial I/\partial z_0^* \) are partial derivatives with respect to the integral parameters.

For the flow regimes under consideration, \( M = 6-10, \) \( Re = 5 \times 10^3-10^8, \) and \( 0 < \alpha < 10^\circ, \) the nondimensional span of the optimal bodies, \( z_0^*, \) increases with increase in the angle of attack and decrease in the volume coefficient, and the