On the Free Convection from a Horizontal Plate

By Keith Stewartson, Durham, England

The free convection of heat from a heated vertical plate in a fluid has been extensively studied in recent years. A review of the work done has been given by Squire [4] and subsequently numerical solutions of the governing equations has been given by Ostrach [2] for a wide range of values of the Prandtl number $\sigma$. The convection takes place in boundary layers originating at the lower edge of the plate. Fluid is drawn into them, is heated and gaining buoyancy moves upwards. On the other hand if the plate is cooled relative to the surrounding fluid the situation is reversed for the boundary layers originate at the top of the plate, and the fluid drawn into them is forced downwards. When the plate is inclined to the vertical there is no change in the flow pattern, since the vertical buoyancy force has a component along the plate which drives the fluid thus generating the boundary layer. However, if the plate is horizontal the buoyancy has no component along its length and the boundary layer, if it exists, must be of a different character.

1) Department of Mathematics, The University.
2) Numbers in brackets refer to References, page 281.
In this paper we shall examine the boundary layer on a horizontal plate showing that if the plate is heated and faces downwards or is cooled and faces upwards a solution can be found which is in moderately good agreement with experiment. On the other hand if the plate is heated and faces upwards or is cooled and faces downwards the boundary layer problem is not properly posed.

The appropriate equations for a heated plate inclined at an angle \( \alpha \) to the horizontal are

\[
\begin{align*}
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} - \rho g \sin \alpha + \mu \frac{\partial^2 u}{\partial y^2}, \\
0 &= -\frac{\partial p}{\partial y} - \rho g \cos \alpha, \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{\mu}{\sigma} \frac{\partial^2 T}{\partial y^2},
\end{align*}
\]

in which \( x \) measures distance along the plate and upwards, \( y \) distance perpendicular to the plate and upwards, \( u, v \) are the components of velocity in these directions, \( \rho \) the pressure, \( \rho \) the density, \( T \) the absolute temperature and \( \mu \) the coefficient of viscosity. In deriving these equations it is assumed that both the viscosity and the difference between the ambient temperature \( T_0 \) and the plate temperature \( T_1 \) are sufficiently small. Let

\[
\rho = \rho_0 - \rho_0 g \gamma \cos \alpha - \rho_0 \rho \gamma \sin \alpha + \rho_0 P,
\]

where \( \rho_0 \) is the constant density of the fluid outside the boundary layer and \( \rho_0 P \) is a constant. Since the boundary layer is thin, it follows from (2) that \( \rho_0 P \), which is the pressure increment inside it, is small. From the equation of state [4] we also have

\[
\rho - \rho_0 = -\beta \rho_0 (T - T_0),
\]

where \( \beta \) is a constant. In particular for a gas \( \beta T_0 = 1 \). The first two equations of motion now reduce to

\[
\begin{align*}
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{\partial P}{\partial x} + \beta g \sin \alpha (T - T_0) + v \frac{\partial^2 u}{\partial y^2}, \\
0 &= -\frac{\partial P}{\partial y} + \beta g \cos \alpha (T - T_0),
\end{align*}
\]

where \( v \) is the kinematic viscosity.

The boundary conditions are that \( u = v = 0, T = T_1 \) at \( y = 0 \) and \( u = 0, T = T_0, P = 0 \) outside the boundary layer or, effectively, as \( y \to \infty \). By themselves these are not sufficient to solve the problem completely since the equations are parabolic and initial profiles of \( T \) and \( u \) must be prescribed as well. If \( \alpha \neq 0 \) the difficulty may be overcome as follows. From (7) \( u \) must be positive if the plate is heated and therefore the boundary layer must originate at the lower edge of the plate where it must contain a zero mass of fluid. Similarly if the plate is cooled it must originate with zero mass at the upper edge of the plate.