UNSTEADY DISSIPATIVE STRUCTURES IN NON-NEWTONIAN FLUID FLOW THROUGH A POROUS MEDIUM

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The nonuniform space-time pressure and velocity distributions in an initially nonempty stratum with constant initial pressure created by pumping a non-Newtonian fluid through the boundary of the stratum are investigated. The injected fluid and the fluid present in the stratum before injection have identical physical properties. The conditions of formation of traveling fronts and localized structures are analyzed as functions of the nonlinearity of the rheological law of the fluid and the injection regime.

Traveling front type self-wave structures in percolating gas and unpressurized liquid flows were studied in [1-3]. As distinct from structures of the traveling wave type, the velocity and the shape of the disturbed region are not constant. As in the problem of heat propagation [4], the principal condition of formation of these fronts is a zero initial state of the medium (initially empty stratum or dry soil [3]). Since the main equations are invariant under translation of the pressure function, the structures considered here are formed against a background of constant initial pressure in the porous medium. Steady-state dissipative structures of the traveling wave type \( P=P(x-Vt) \) \((\infty < t < 0)\) in non-Newtonian flow through porous media were studied previously in [5].

1. SELF-SIMILAR SOLUTIONS FOR GIVEN VARIATION OF THE PRESSURE ON THE BOUNDARY

The equations of one-dimensional plane-parallel flow of a power-law non-Newtonian fluid through a porous stratum have the form [5]:

\[
\frac{\partial P}{\partial t} - \frac{\partial U}{\partial x} = -\Pi \left( \frac{U}{\lambda} \right)^n, \quad n > 0
\]

Here, \( P \) is the pressure, \( U \) is the filter velocity, and \( \beta \) is the expansion capacity of the saturated stratum; \( \Pi \) and \( \lambda \) are the characteristic values of the pressure gradient and the filter velocity, respectively; \( n \) is a number characterizing the non-Newtonian properties of the fluid. For a pseudoplastic fluid \( n < 1 \), for a dilatant fluid \( n > 1 \).

For Eqs. (1.1) we will consider the following boundary-value problem of fluid injection into a reservoir with constant initial pressure \( P_0 \) using a power law of pressure variation on the reservoir boundary:

\[
U(x, 0) = 0, \quad P(x, 0) = P_0 = \text{const}
\]

\[
P(0, t) = P_0 + A t^{-m}, \quad A = \text{const}, \quad t > 0, \quad m > \max(-1, -n/2)
\]

In this case the functions \( P \) and \( U \) decrease into the reservoir, i.e., the corresponding gradients \( P_x \) and \( V_x \) are negative.

The self-similar solution of the problem (1.1) - (1.3) will be sought in the form:

\[
P = P_0 + A t^{-m} \psi(\xi), \quad U = r(t) f(\xi), \quad \xi = x/\phi(t)
\]

The functions \( \psi(\xi), f(\xi), \phi(t) \), and \( r(t) \) are to be determined.

Using dimensional analysis [6], we can find the functions \( \phi(t) \) and \( r(t) \) in the form:

\[
\phi(t) = (A^{1-n} \Pi^{-1} \lambda^n)^{1/(1-n)} t^{(m(1-n)-n)/(n+1)}
\]

In the linear case \( n = 1 \) Eq. (1.10) reduces to the Hermite equation and for \( 1.11 \) its solution has the form \[ 7J: \]

\[
V(E) = 2y_{1, n - p} \exp(-E^2/4) H_{1/2}(\sqrt{2})
\]

\[ (*) \]

Here, \( H(x) \) is the Hermite function, and \( \Gamma(x) \) is the gamma-function. From (1.12) and (1.5) - (1.8) we can obtain the complete solution of the problem of the pressure \( e \) and velocity field distribution for the elastic flow regime. In this case, along with the condition \( n = 1 \), we must replace the ratio \( \Pi/\lambda \) by the ratio \( k/\mu \) in formulas (1.7) and (1.6) \( k \) is the permeability and \( \mu \) is the viscosity.

In the case \( n \neq 1 \) Eq. (1.10) admits the group of transformations

\[
\delta = a_{1+n}^{(1-n)^n}, \quad (1+n) = a_{1+n}^{(1-n)^n}
\]

where \( a \) is an arbitrary positive constant.

We will reduce the order of (1.10), using the following substitution:

\[
\eta = \ln \xi, \quad \psi(\xi) = \xi^{(1-n)/(1-n)} F(\eta), \quad \frac{dW}{dF} = \frac{C_1 W - C_2 W^{1/n} - mnF}{W^{(1-n)/n} (W - C_3 F)}
\]

\[ (*) \]

\[
W = \frac{dF}{d\eta} + C_2 F; \quad C_3 = \frac{1 + n}{1 - n}; \quad C_1 = \frac{mn(1 - n) + n^2}{1 + n}; \quad C_2 = \frac{2n}{1 - n}
\]

We now go over to the consideration of concrete problems.

2. FLUID INJECTION AT CONSTANT PRESSURE \( m = 0 \)

Equations (1.7) and (1.8) can be reduced to the single equation

\[
\frac{df}{d\xi} + \frac{n}{n + 1} \xi f^n = 0
\]

The solution of (2.1) has the form:

\[
\xi^2 = \frac{2(1 + n)}{n(1 - n)} [ f^{1 - n}_{0} - f_{0}^{1 - n}], \quad f \leq f_{0}
\]

where \( f_{0} = f(0) \) is a constant remaining to be found. The function \( f(\xi) \) determined from (2.3) decreases monotonically and possesses the following properties: for \( n \geq 1 \) \( f \to 0 \) as \( \xi \to \infty \); for \( n < 1 \) \( f \to 0 \) as \( \xi \to a < \infty \), where

\[ (*) \]