Thermal Stresses in Viscoelastic Structures

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1. Introduction

In the standard texts on the theory of structures, more space is allotted to trusses than seems warranted by their role in modern structural engineering. This is done because the discussion of the mechanical behavior of trusses requires but a minimum of mathematics and thus provides an excellent opportunity for developing the student's understanding of the mechanical behavior of structures in general. In a similar manner, the influence of various mechanical effects on structural behavior is most readily investigated for trusses. As a rule, the extension of the results of such an investigation to other types of structures does not present any difficulties.

In recent papers Freudenthal [1], [2] used the simple truss shown in Figure 1 as a convenient structural model for the discussion of inelastic thermal stresses. The state of loading of this simply indeterminate structure was supposed to be specified by a single parameter, the load intensity $P$, and only the bar $OB$ was supposed to exhibit inelastic behavior; the two other bars were assumed to be perfectly elastic.

Even so simple a model may reflect many characteristic traits, but it cannot be expected to reveal all these traits. A more general investigation of thermal stresses in inelastic trusses seems therefore desirable.

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1) The results presented in this paper were obtained in the course of research sponsored by the Ballistic Research Laboratories of Aberdeen Proving Ground under Contract DA-19-020-ORD-788.

2) Brown University.

3) Numbers in brackets refer to References, page 237.

4) For perfectly plastic structures, for instance, this intuitive statement is confirmed by the work of Prager and Symonds [3], [4].
The present paper is concerned with the thermo-mechanical behavior of a statically indeterminate truss that consists of Maxwell bars. The effects of typical variations of loads and temperatures on the stresses in the truss are discussed. While the actual relation between creep rate and stress in structural metals is less simple than the linear relation stipulated for a Maxwell material, the structural model considered in the following may be expected to indicate the general effect of creep on the stresses in indeterminate structures.

2. Fundamental Relations

The principle of virtual displacements is the only tool of structural theory used in the following discussion. For a truss, this principle involves, on the static side, loads $P$ and bar forces $S$ that are in equilibrium with these loads, and, on the kinematic side, joint displacements $u$ and bar elongations $\lambda$ that result from these displacements. The principle of virtual displacements then asserts that

$$\sum S \lambda = \sum P \cdot u,$$

where the sum on the left includes all bars and that on the right all joints of the truss, the dot being used to indicate the scalar product.

The discussion of the mechanical behavior of highly indeterminate structures is greatly facilitated by the introduction of orthonormal states of stress (see, for instance, [5]). Consider two states of stress in a given truss, and let $S'$ and $S''$ denote the corresponding forces in a generic bar. If $l$ denotes the length of this bar, $A$ its cross-sectional area, $E$ its modulus of elasticity, and $c = l/(AE)$ its elastic compliance, the two states of stress are said to be orthogonal if

$$\sum c S' S'' = 0,$$

where the sum includes all bars of the truss. If moreover

$$\sum c S'^2 = 1, \quad \sum c S''^2 = 1,$$

the two states of stress are said to be orthonormal. Two groups of loads acting on a truss are called orthonormal, if the elastic bar forces corresponding to them are orthonormal. These elastic forces must be computed by considering only the elasticity of the bars but neglecting all other rheological properties, e.g. viscosity or plasticity.

A system of loads that depends on $m$ parameters is most conveniently treated as a linear combination of $m$ orthonormal groups of loads. The coefficient $p'_i$ of the $i$-th group will be called the intensity of this group in the linear combination. If $S'_i$ is the elastic force caused in a generic bar by the unit intensity of the $i$-th group, the elastic response of this bar to an arbitrary