Relativistic Optics of Nondispersive Media

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The relativistic optics of the nondispersive media endowed with the metric \(g_\alpha(x)\) [Eq. (1.6)] and with a nonlinear connection [Eq. (1.2)] is studied. The \(d\)-connection [Eqs. (3.3)–(3.4)] relates the conformal and projective properties of the space-time. A post-Newtonian estimation for the metric \(g_\alpha(x)\) is also given. It is shown that the solar system tests impose a constraint [Eq. (4.20)] on a combination of the post-Newtonian parameters describing the model.

1. INTRODUCTION

In a previous paper,\(^{(1)}\) Miron and Tavakol have studied a new possibility to realize the EPS axiomatics (Ehlers, Pirani, and Schild)\(^{(5)}\) of general relativity using models more general than the classical ones. The model given by these authors is based on a generalized Lagrange space \(GL^n\) endowed with the conformal metric (axiom \(a_1\)):

\[
g_\gamma(x, y) = e^{2\sigma(x, y)} g_\gamma(x) \tag{1}\]

where \(\sigma(x, y)\) is a function defined over the tangent bundle TM of a \(C^\infty\)-dimensional real manifold \(M\), and \(g_\gamma(x)\) is a pseudo-Riemannian metric on the manifold \(M\). The generalized Lagrange space \(GL^n\) is endowed with a nonlinear connection (axiom \(a_2\))\(^{(1,9,10)}\):

\[
N^i_j(x, y) = \left\{ \begin{array}{l}
i \\
jk \\
\end{array} \right\} y^k \tag{2}\]

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where \( \{ j_k \} \) are the Christoffel symbols of the metric \( \gamma^j_y(x) \). If the function \( \sigma(x, y) \) is chosen in a suitable way, then the metric (1) implies new properties of the EPS axiomatics. Indeed, considering \( \sigma(x, y) \) of the form

\[
\sigma(x, y) = \frac{\alpha}{2} \left( 1 - \frac{1}{n^2(x, y)} \right)
\]

(3)

where \( \alpha \) is a positive constant and \( n = n(x, V(x)) \) is the index of refraction defined on the dispersive medium \( M = (M, V(x), n(x, V(x))) \), then (1) becomes a metric that is suitable for the study of physical properties of the mentioned dispersive medium \( M \). We can describe the relativistic optics as a geometrical study of the pair \( \{ g_y(x, V(x)), N^j(x, V(x)) \} \), where

\[
g_y(x, V(x)) = e^{2\sigma(x, V(x))} \gamma^j_y(x)
\]

(4)

\( N^j \) is given by (2) and \( \sigma \) has the form (3) with \( y^k = V^k(x) \). Then, the geometrical model of the relativistic optics appears as a generalized Lagrange space \( GL^n(TM, g_y(x, y)) \) endowed with the nonlinear connection \( N^j \) and this theory must be restricted to the section \( S_\nu: M \rightarrow TM \) of the natural projection \( \pi: TM \rightarrow M \), locally given by

\[
S_\nu: \begin{cases}
x^i = x^i \\
y^i = V^i(x), \quad \forall x \in M
\end{cases}
\]

(5)

A special case is that in which the medium \( M \) is nondispersive, i.e., the index of refraction does not depend on the velocity \( V(x) \).

In this paper we will study the relativistic optics of the nondispersive medium \( M \) endowed with the metric

\[
g_y(x, V(x)) = e^{2\sigma(x, V(x))} \gamma^j_y(x), \quad \sigma(x, y) = \frac{\alpha}{2} \left( 1 - \frac{1}{n^2(x, y)} \right)
\]

(6)

and with the nonlinear connection \( N^j(x, y) = \{ j_k \} y^k \). We will interpret the metric (6) as a generalized Lagrange metric of a generalized Lagrange space \( GL^n \) endowed with the nonlinear connection (2). In this way, we realize the EPS conditions by the axioms:

(a_1) The space-time has the conformal structure given by (6);

(a_2) The space-time has the projective structure given by the autoparallel curves of the nonlinear connection (2).

These structures are determined from a physical point of view by the light propagation and, respectively, by the free falling without rotation of the test particles. Consequently, the structure of this space is not strictly a