MONTE CARLO (IMPORTANCE) SAMPLING WITHIN A BENDERS DECOMPOSITION ALGORITHM FOR STOCHASTIC LINEAR PROGRAMS

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Abstract

This paper focuses on Benders decomposition techniques and Monte Carlo sampling (importance sampling) for solving two-stage stochastic linear programs with recourse, a method first introduced by Dantzig and Glyrm [7]. The algorithm is discussed and further developed. The paper gives a complete presentation of the method as it is currently implemented. Numerical results from test problems of different areas are presented. Using small test problems, we compare the solutions obtained by the algorithm with universe solutions. We present the solutions of large-scale problems with numerous stochastic parameters, which in the deterministic formulation would have billions of constraints. The problems concern expansion planning of electric utilities with uncertainty in the availabilities of generators and transmission lines and portfolio management with uncertainty in the future returns.

1. Introduction

A stochastic linear program is a linear program whose parameters (coefficients, right-hand sides) are uncertain. The uncertain parameters are assumed to be known only by their distributions. This means that the values of some functions are numerical characteristics of random phenomena, e.g. mathematical expectations of functions dependent on decision variables and random parameters.

Suppose a function \( z = E C(V) \) is an expectation of a function \( C(v^\omega) \), \( \omega \in \Omega \). \( V \) is a random parameter which has outcomes \( v^\omega \). \( \Omega \) is the set of all possible random events. It can be finite, infinite, discrete or continuous. In the continuous case, the computation of the expected value requires the solution of the integral:

\[
EC(V) = \int C(v^\omega)P(d\omega),
\]

with \( P \) being the probability measure.

In a general case, \( V \) would consist of several components, e.g. \( V = (V_1, \ldots, V_h) \) with outcomes \( v^\omega \), which we also will denote only by lower case letters, e.g. \( v = (v_1, \ldots, v_h) \) and \( p(v^\omega) \) alias \( p(v) \) would denote the corresponding density.
function. We assume the components of $V$ to be independent. In addition, we will construct $\Omega$ by crossing the sets of outcomes $\Omega_i$ for vector entry $v_i$, $i = 1, \ldots, h$ as

$$\Omega = \Omega_1 \times \Omega_2 \times \ldots \Omega_h.$$ 

In this case, the above-mentioned integral takes the form of a multiple integral:

$$EC(V) = \int \cdots \int C(v)p(v)dv_1 \ldots dv_h.$$ 

In the case of $\Omega$ being discrete and finite, the expectation can be computed by a multiple sum:

$$EC(V) = \sum_{v_1} \cdots \sum_{v_h} C(v)p(v).$$ 

The main difficulties in stochastic linear programming deal with the evaluation of the multiple integral or the multiple sum. The numerical computation of the expectation requires a large number of function evaluations, and each function evaluation means a linear program to be solved. Different approaches attack this problem, e.g. Birge [3], Birge and Wets [5], Birge and Wallace [4], Frauendorfer [12], Frauendorfer and Kall [13], Ermoliev [10], Higle and Sen [18], Kall [20], Pereira et al. [26], Rockafellar and Wets [27], Ruszczynski [28], Wets [30], and others. See Ermoliev and Wets [11] for references. We follow the concept of Dantzig et al. [8] and Dantzig and Glynn [7].

2. Two-stage stochastic linear program

An important class of models concerns dynamic linear programs. Variables which describe activities initiated at time $t$ have coefficients at time $t$ and $t + 1$. Deterministic dynamic linear programs appear as staircase problems. The simplest staircase problem is that with two stages: $X$ denotes the first, $Y$ the second-stage decision variables, $A, b$ represent the coefficients and right-hand sides of the first-stage constraints, and $D, d$ concern the second period constraints together with $B$ which couples the two periods. $c, f$ are the objective function coefficients.

In the deterministic case, $c, f, A, b, B, D, d$ are known with certainty to the planner. In the stochastic case, the parameters of the second stage are not known to the planner at time $t = 1$, but will be known at time $t = 2$. At time $t = 1$, only the distributions of these parameters are assumed to be known. The second-stage parameters can be seen as random variables which obtain certain outcomes with certain probabilities. We denote a certain outcome of these parameters with $\omega$ and the corresponding probability with $p^\omega$, $\omega \in \Omega$, the set of possible outcomes.