Mössbauer NMR double resonance in $^{57}\text{Fe}$. 
Coupling between quadrupole split states

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In order to describe the interaction of a nucleus (in a static electric field gradient) with a radiation field, we have introduced the concept of “dressed nucleus”. The eigenvalues of its Hamiltonian are calculated, which leads to expressions for the different energies of $\gamma$-rays produced by spontaneous emission. We have calculated these energies as well as their relative probabilities in the case of $I_e = 3/2^- - I_g = 1/2^-$ M1 transitions. We have shown that a Mössbauer spectrum using as a source an ensemble of “dressed nuclei” and a single line absorber consists of six lines: two lines of the original quadrupole doublet and four sidebands, each of them having the same intensity.

1. Introduction

We have introduced the concept of “dressed nuclei” in ref. [1] in order to study the interactions of an ensemble of nuclei in the presence of a static magnetic field with a radiation field. The idea is to consider the global system nucleus-static magnetic field-radiation field as one quantum system in the Schrödinger representation. We have treated the resonant coupling of the excited state nuclear Zeeman sublevels (nuclear spin $I_e = 3/2^-$) by a radiation field. When $\gamma$-rays are produced due to spontaneous emission (nuclear ground state spin $I_g = 1/2^-$), we have shown that 24 different energies occur. We have calculated these energies as well as the relative intensities of the transitions.

In this paper, we will investigate the aspects of the Mössbauer spectrum when a radiation field couples the components of quadrupole split states.

2. Eigenvalues of the Hamiltonian

It is well known that a nucleus with spin $I$ and quadrupole moment $Q$ in a static axially symmetric field gradient $V_{zz} = eq$ is described by the Hamiltonian

$$H_N = H_{N,0} + H_Q,$$

(1)
with $H_{N,0}$ the Hamiltonian of the free nucleus and

$$H_Q = \frac{e^2 q Q}{\hbar^2 4I(2I - 1)} (3I_z^2 - I^2). \quad (2)$$

The eigenvalues $E_N$ of $H_N$ are

$$E_N = E_{N,0} + \frac{e^2 q Q}{4I(2I - 1)} [3m_I^2 - I(I + 1)]. \quad (3)$$

with $E_{N,0}$ the free nuclear energy and $-I \leq m_I \leq I \ (I \geq \frac{1}{2})$. Equation (3) shows that the $(2I + 1)$-fold energy degeneracy is partially lifted by the Q.I. A state with $I = \frac{3}{2}$ splits into two sublevels, a state with $I = \frac{1}{2}$ is not split. The normalized eigenstates of $H_N$ are denoted by $| \frac{3}{2}, m_I \rangle$.

Let us now consider a radiation field with only one excited mode and having a fixed number of photons $n$ before the interaction takes place. Later, we will consider a more realistic coherent field. This field has a Hamiltonian $H_f$ given by

$$H_f = \hbar \omega (a^+ a + \frac{1}{2}), \quad (4)$$

whose eigenvalues are

$$E_f = \hbar \omega \left( n + \frac{1}{2} \right), \quad (5)$$

with $n$ integer $\geq 0$. The corresponding normalized eigenvectors of $H_f$ are denoted $| n \rangle$. When there is no interaction between the nucleus and the radiation field, the Hamiltonian $H_0$ of the global system nucleus-electric field gradient-radiation field is simply

$$H_0 = H_N + H_f. \quad (6)$$

The eigenvalues $E_0$ of $H_0$ are sums of the eigenvalues of $H_N$ and $H_f$

$$E_0 = E_{N,0} + \frac{e^2 q Q}{4I(2I - 1)} [3m_I^2 - I(I + 1)] + \hbar \omega \left( n + \frac{1}{2} \right). \quad (7)$$

The corresponding eigenvectors of $H_0$ are direct products of the eigenvectors of $H_N$ and those of $H_f$: $| I, m_I \rangle \otimes | n \rangle$. From now on, we will choose $I = 3/2^-$, $V_{zz} > 0$ and $Q > 0$.

Let us consider the two orthonormal states $| \frac{3}{2}, -\frac{1}{2} \rangle \otimes | n \rangle$ and $| \frac{3}{2}, -\frac{3}{2} \rangle \otimes | n - 1 \rangle$. They span a two-dimensional linear space, denoted by $M_{\frac{3}{2}, -\frac{1}{2}, n} : | \frac{3}{2}, -\frac{1}{2} \rangle \otimes | n \rangle \otimes | n - 1 \rangle$. The corresponding to the energy $E_1$ given by

$$E_1 = E_{N,0} - \frac{e^2 q Q}{4} + \hbar \omega \left( n + \frac{1}{2} \right) \quad (8)$$