Continuous-Time Markov Processes as a Stochastic Model for Sedimentation

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Markov processes with a continuous-time parameter are more satisfactory for describing sedimentation than discrete-time Markov chains because they treat sedimentation as a natural process that happens continuously (i.e., which is unbroken in time). They also avoid certain technicalities that arise in discrete time—namely, the choice of a time unit. Finally, they yield not only the same information as a discrete-time analysis, but also give information about the distribution of the thicknesses of the lithologies.

KEY WORDS: Markov process, lithology, Markov renewal process.

INTRODUCTION

In the past, various researchers (e.g., Gingerich 1969; Ethier, 1975; Powers and Easterling, 1982; Carr, 1982), have used the theory of discrete-time Markov chains to describe the structure of a sequence of lithologies. A Markov process is a stochastic process, meaning a sequence of random events, in which the only information useful for predicting the state of the sequence at time $n$ contained in the history of the process (i.e., the sequence of states visited before time $n$) is the last state observed:

$$P(Y_n = j | Y_{n-1} = i, \ldots, Y_0 = i_0) = P(Y_n = j | Y_{n-1} = i) = q_{ij}$$

Here $n$, a natural number, is a discrete-time parameter, $Y_n$ is the random variable describing the state the process occupies at time $n$, $i$, $i_0$ to $i_{n-1}$ and $j$ are elements of the state space (e.g., the rock types of a stratigraphic column). The matrix $Q$ with entries $q_{ij}$ is called the transition matrix of the process. It gives the probability $q_{ij}$ of a transition from state $i$ to state $j$.

This Markov chain is called time-homogeneous because the transition matrix $Q$ does not depend on the time parameter $n$. This means the stochastic structure is the same throughout the time of the evolution of the system. Ho-
Mogeneity within the observed column is an essential assumption for the theory described here, although in reality it may not always be fulfilled.

Processes with a "memory" longer than the one considered—in particular, processes where the knowledge of a greater but fixed part of the history of the process, here, the sequence of lithologies in the stratigraphic column, is helpful—are so-called multistage Markov processes. Although processes of such a type will not be discussed in this paper, the theory can easily be extended to that case. The statistical analysis of multistage Markov chains has been treated by Chatfield (1973).

One of the problems arising in the modeling of sedimentary sequences as a discrete-time Markov chain is the definition of one unit, either of time or of the thickness of a lithology. Ethier (1975) showed that the choice of a fixed unit leads to transition matrices with overly large frequencies on the diagonal \((i = j)\), which means that "changes" from a rock type to itself tend to be over-represented.

Another possibility is to count the number of transitions from one rock type to another, disregarding the thickness of the layers. This approach leads to the so-called embedded Markov chains, which have zeroes on the diagonal. Those zeroes are structural because they do not reflect probabilities but are a result of the mathematical model. A statistical test for the hypothesis of independence in the stochastic process (i.e., the independence of the process from its history, given the structural zeroes) has been described by Goodman (1968), and was introduced in the geological literature by Powers and Easterling (1982).

Although the model described in this paper is essentially different from previous ones, this test will also be of use for the statistical analysis of continuous-time Markov processes.

A third method to attack this problem has not as yet been discussed in the literature, and that is to use continuous-time Markov processes as a model. These are processes with a continuous-time parameter which also have the Markovian property that, given the present state, the future is independent of the past. "Continuous" does not mean uniform, but unbroken, and it only assumes that sedimentation does not happen instantaneously. "Time" in our context will mean the thickness of a sedimentary segment. For this analysis to be valid, one has to assume that the sedimentation has reached a position where subsequent erosion is not likely.

The thickness of a segment of the stratigraphic column is a random variable. If the continuous-time Markov model fits, then the theory of Markov processes show that the sequence of thicknesses of every rock type is "memory-less" (i.e., it is a sequence of independent exponential random variables. For a mathematical derivation of this fact, see Karlin and Taylor (1975).

To establish that a stratigraphic column follows a continuous-time Markov process, two steps are necessary. In the first step, one shows that the sequence of rock types follows a discrete Markov chain. This step is exactly the proce-