RELAXATION OF BOUND ELECTRONS IN ELASTIC COLLISIONS WITH INFINITELY HEAVY PARTICLES

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The recombination relaxation of bound electrons in collisions with infinitely rigid, infinitely heavy spheres is considered. The relaxation mechanism is due to a change in the electron scattering from the spheres. The distribution functions of bound electrons and the recombination time due to that mechanism are obtained. The characteristics of the relaxation are the same as for the Thompson recombination mechanism (caused by energy transfer from the electron to a third particle) to within replacement of the mass of the third particle by the ion mass.

INTRODUCTION

Two elastically colliding particles cannot recombine, i.e., they cannot go from the free state to a bound state. In the center-of-inertia system the momenta of the particles are in opposite directions and equal in value. After the elastic collision, because of the conservation of energy and momentum of the system, the center-of-inertia velocity does not change and the momenta of the particles remain equal in value but in opposite directions (they only change their general orientation). For recombination to occur it is necessary that either internal degrees of freedom of the colliding particles are excited or there is interaction with a third object (particles, electromagnetic field, wall).

Since Thompson [1], the mechanism of three-particle recombination of charged particles in a fairly rarefied gas has been considered as being the result of energy transfer from one of the interacting charged particles to a third particle [2-6] (three-particle or three-body recombination). Having lost part of its energy in a collision with the third body, the charged particle is bound with the charge in whose field the collision occurred. Generally speaking, the recombination is not an elementary act but a process, i.e., a combination of elementary acts described by kinetic equations (see, e.g., [6-8]).

Ion-electron recombination often can be described on the basis of the equation of diffusion along the energy axis, since the energy transfer from the electron to the third body usually occurs in small portions. In the collision of an electron with atoms, the energy transfer is small because the masses of the electron and atom differ so much [2, 6]; if an electron is the third body [3-5, 9], the diffusion nature of the energy relaxation is due to the long-range character of the Coulomb forces.

Studies of the properties of a classical Coulomb plasma, which have been carried out in recent years by simulation of multiparticle dynamics from first principles (MPD method) [10-12], have heightened interest in the analysis of three-particle recombination mechanisms. The equation [3] for the energy axis diffusion coefficients has been made substantially more exact and has been used to calculate both the recombination coefficient and the electron distribution in the region of negative energies [5, 9].

For a comparison with the results of MPD simulation, it is of interest to consider the details of recombination with the participation of various hypothetical third particles. In [11-14], for example, we analyzed the recombination relaxation due to collisions with two-level neutral particles. In this paper we consider relaxation due to collisions of electrons with infinitely heavy rigid spheres.
1. DIFFUSION EQUATION

The Fokker–Planck equation, which describes energy axis diffusion, is taken in the form, as usual [5, 9],
\[
\frac{\partial f}{\partial t} = -\frac{\partial \Gamma}{\partial \varepsilon}; \quad \Gamma = A - \frac{\partial (B f)}{\partial \varepsilon} = A - B \frac{\partial f}{\partial \varepsilon}.
\]
(1)

Here \( \varepsilon \) is the total electron energy, \( A = \lim_{\tau \to 0} \frac{\Delta \varepsilon}{r} \) and \( B = \lim_{\tau \to 0} \frac{\Delta \varepsilon^2}{2r} \) are coefficients that characterize the mobility and diffusion along the energy axis, \( \tilde{A} = A - \frac{\partial B}{\partial \varepsilon} \) is the modified mobility coefficient, and \( \Gamma \) is the energy axis flux (for recombination \( \Gamma > 0 \)).

Moreover, we use the ordinary quasisteady approximation \( \frac{\partial f}{\partial t} = 0, \Gamma = \text{const} \) and normalize the distribution function to unity, \( \int f(\varepsilon) d\varepsilon = 1 \). The characteristic recombination time is given by
\[
dN_e/dt = -N_e/\tau_{rec} = \Gamma N_e.
\]

When considering the relaxation due to pair collisions, the coefficients of the Fokker–Planck equation should obey the detailed balance relation
\[
A(\varepsilon)f_*(\varepsilon) = B(\varepsilon)df_*/d\varepsilon \quad \text{or} \quad A(\varepsilon)f_*(\varepsilon) = d(B(\varepsilon)f_*)/d\varepsilon,
\]
(2)

\[f_*(\varepsilon) = g(\varepsilon)\exp(\varepsilon/T_e)
\]
(3)

is the Boltzmann distribution, \( T_e \) is the electron temperature,
\[
g(\varepsilon) = \frac{1}{\pi^{1/2}} \frac{\varepsilon}{e^{2}} \left( e - \varepsilon \right) \exp\left( \frac{\varepsilon}{\epsilon} \right) \int \frac{\varepsilon}{e^{2}} d\varepsilon = \text{const} \frac{\varepsilon^{1/2}}{T_e} \left| \frac{\varepsilon}{T_e} \right|^{1/2} \exp\left( \frac{\varepsilon}{\epsilon} \right)
\]
(4)

is the energy density of states (integration is carried out over the coordinates \( n_i \) and the velocities \( v_i \) of the test electron), \( N_e = N_i \) is the ion density, and \( \delta = 2e^2N_i/T_e^3 \) is a parameter that characterizes an ideal plasma.

2. RELAXATION DUE TO ELECTRON COLLISIONS WITH INFINITELY HEAVY ELASTIC SPHERES

Variation of the Binding Energy. At first glance, collisions with an infinitely heavy particle cannot lead to energy axis relaxation since the kinetic energy of an electron does not change. In the given case, however, the recombination mechanism can be slightly different from the Thompson mechanism. The point is that when an electron collides with a third body, there is no momentum conservation for the electron–ion pair. Although the total energy of the system does not change, the internal energy of the particle pair (i.e., the energy in the center-of-inertia system) and the energy of the motion of the particles as a whole is redistributed. In other words, in the Thompson model, recombination occurs because of nonconservation of energy for a pair of charged particles when they collide with a third particle, and in the case under consideration recombination is tied to the nonconservation of momentum of the pair.

Suppose that \( v_1, v_2, m_1, m_2 \) are the velocities and masses of particles 1 and 2. For the center-of-mass velocity \( V \) and the relative particle velocity we have
\[
V = (\mu / m_1) v_2 + (\mu / m_2) v_1, \quad u = v_2 - v_1.
\]
where \( \mu = m_1 m_2/(m_1 + m_2) \) is the reduced mass.