YANG—MILLS FIELDS WITH EXTERNAL CURRENT

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A procedure for quantizing a non-Abelian gauge theory with Lagrangian

\[ L = \frac{1}{4} F_{\mu \nu}^a F^{\mu \nu}_a + J_a^\mu V^{a \mu} \]

near a nontrivial classical solution is considered. The theories are classified with respect to the external current. The gluon propagator in a model spherically symmetric non-Abelian field is constructed and investigated.

1. INTRODUCTION

This paper is devoted to application of the method of canonical quantization to a particular field theory model. The method of canonical quantization was first formulated by Dirac in 1925 [1]. In 1950, Dirac [2] extended it to theories with constraints, creating the foundations of the so-called generalized Hamiltonian formalism. After the appearance of non-Abelian gauge theories in 1954 [3], the method found nontrivial application. Although already in 1967 Faddeev and Popov [4] specified the form of the generating functional of the Green's functions for Yang—Mills fields, it was only in 1969 that Faddeev [5] fully clarified the situation having considered the problem from the Hamiltonian point of view. The book [6] is devoted to an exposition of the modern understanding of the method of canonical quantization.

Conceptually, the present paper is close to studies on the quantization of gauge theories in the neighborhood of external fields of various configurations. The first study we must mention is that of Savvidi [7] (with additions by Nielsen and Olesen [8], and also Skalozub [9]). Later, other external fields were proposed [10], in particular the so-called 3k field (see, for example, [11], where the theory is considered at nonzero temperature). A distinctive feature of these fields is that they do not satisfy the Yang—Mills equations, and therefore the canonical procedure for constructing perturbation theory on the background of the external field is inapplicable. Brown and Weisberger [10] were the first to write down a modified Lagrangian of a Yang—Mills field with external current. As the starting point for the construction of a theory, Kabo and Shabad considered such a Lagrangian [12]. In this paper, we analyze in detail the Hamiltonian structure of a theory with external current. It turns out that a Lagrangian that at the first glance seems harmless leads (depending on the structure of the external current) to four different Hamiltonian theories that differ, among other things, in the number of physical degrees of freedom. In the present paper, we investigate in most detail non-Abelian external fields without zeroth component (the 3k field belongs to this type).

The quantization of a gauge theory in the neighborhood of an external field that is a solution of the classical Yang—Mills equations,

\[ \nabla_{\mu} F_{\mu \nu}^{ab} = 0, \]

was considered in detail by Aref'eva, Slavnov, and Faddeev [13]. We emphasize once more that the aim of our paper is to quantize a gauge theory in the neighborhood of an external field that satisfies the classical Yang—Mills equations with nonzero current:

\[ \nabla_{\mu} F_{\mu \nu}^{ab} = -J_a^\nu. \]

Usually, in a field theory such as QED, the various field configurations have been completely characterized by two invariants: \( F_{\mu \nu}^\rho F^{\mu \nu}_{\rho} \) and \( F_{\mu \rho}^\rho \) (where \( F_{\mu \nu}^\rho = \varepsilon_{\mu \rho \sigma} F^{\sigma \nu} \)), and the fields themselves have satisfied Maxwell’s equations that could have on their right-hand side a current \( J \) (on which was imposed the continuity condition corresponding to charge conservation in the system). Since in QED any current can be specified “by hand,” problems with construction of the fields did not arise, and it was assumed that all fields could be modeled. This is a first circumstance that creates a significant difference between QED and QCD, in which the problem of modeling fields is not so trivial, since as yet (and, perhaps, altogether by virtue of


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the confinement hypothesis) we cannot specify the currents on the right-hand side of the Yang—Mills equations and, accordingly, the field configurations. A no less important circumstance is the existence of nine invariants in Yang—Mills theory [10, 14, 15], which, as will be shown in this paper, not only do not fix the field structure (on account of the Wu—Yang ambiguity [16]) but also, corresponding to solutions of the Yang—Mills equations with current on the right-hand side, can belong to different gauge-inequivalent physical theories. This last circumstance is very important, since it raises the question of what is the physical realization that corresponds to a particular configuration of a non-Abelian field. The answer to this question does not reside in the specification of the vector potentials of the field but in the precise specification of the total Lagrangian in the framework of which these field configurations arise as solutions of the classical field equations of motion. The posed problem can be solved by carrying out the procedure of canonical quantization of one of the field theory models with current.

In the paper, we consider the model of a vector field \( V_a(x) \) with the Lagrangian (see [12])

\[
L = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + J_a^\mu V_a, \quad a = 1, 2, 3, \quad (*)
\]

where

\[
F_{\mu\nu}^a = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + g\varepsilon_{abc} V_\mu^b V_\nu^c.
\]

We consider the problem of the quantization of this model in an external field, i.e., near a nontrivial solution \( A_\mu^a(x) \) of the classical Lagrange equations. The current \( J \) is related to the field \( A \) by the equation

\[
\varepsilon_{\mu\nu\rho} F_\nu^a = -J_\rho^a
\]

(in quantities with a bar, the letter \( V \) will be replaced by the letter \( A \)).

The aims of this paper are: 1) To implement the procedure of consistent quantization of the model \((*)\); 2) to investigate the properties of the gluon propagators in the 3\( \lambda \) field.

A few words about the exposition that follows. Section 2 is devoted to the construction of the generalized Hamiltonian formalism for the model \((*)\). We shall find that there arises naturally a classification of theories in accordance with the external current, and this has four cases. For one of them (we call it the “simple” case and consider it in detail in Sec. 2A), we make a careful construction of the Hamiltonian formalism; for the remainder, we give only the results.

The “simple” case is quantized in Sec. 3. We there analyze the free theory and construct physical variables for it explicitly. We obtain an expression for the generating functional of the Green’s functions.

In Sec. 4, this generating functional is expanded in perturbation theory. The equations for the propagators are solved (in general form). Section 5 is devoted to the explicit calculation of the propagators in the 3\( \lambda \) field.

Notation. One notation has already been used — letters with a bar above them means replacement of the field \( V \) by the external field \( A \). Besides the full notation (of the type \( V_a^\mu \)), we also make wide use of notation in which the isotopic indices are omitted (\( V_\mu, F_{\mu\nu} \... \)). Isotopic vectors and isotopic operators can be distinguished by the context. Thus, to the expression \( \varepsilon_{\mu\nu\rho} F_\nu^a \) there corresponds simply \( \varepsilon_{\nu\rho} F_\rho^a \) and to the “scalar product” \( \varepsilon_{\mu\nu} M_{\mu\nu} \) there corresponds \( \varepsilon M_\gamma \). Special notation is adopted for operators corresponding to vectors — they are identified by “hats”:

\[
(\hat{A})_{ab} = g\varepsilon_{acb} A_c.
\]

Some specific equations now:

\[
\nabla_\mu = \partial_\mu + \hat{V}_\mu, \quad F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + \hat{V}_\mu V_\nu.
\]

For operators of multiplication

\[
\hat{A}\hat{B} - \hat{B}\hat{A} = (\hat{A}\hat{B}).
\]

All operators (even integral operators) are expressed as multiplication operators.

2. GENERALIZED HAMILTONIAN FORMALISM

A. Generalized Hamiltonian Formalism in the (“Simple”) Case \( J_0 = 0, \partial_0 J_k = 0, J_\mu^a \neq J_k^b \mu \)

For the model \((*)\), we shall construct the Hamiltonian formalism in accordance with [6]. As is well known, to do this it is necessary to introduce variables \( F_\mu^a \) conjugate to \( V_\mu^a \), i.e., to specify the fundamental Poisson bracket

\[
\{ F_\mu^a(x, t), V_\nu^b(\vec{y}, t) \} = g_{\mu\nu}\delta^{ab}\delta(\vec{x} - \vec{y}).
\]

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