SUPERCONDUCTIVITY IN TWO-BAND SYSTEMS WITH A SMALL CONCENTRATION OF CARRIERS

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Superconductors with two characteristic features of high-temperature superconducting materials, i.e., a small concentration of charge carriers and overlapping energy bands on the Fermi surface, are considered. At \( T = 0 \), the order parameters \( \Delta \) and the chemical potential \( \mu \) for the Bose condensate of local pairs (\( \mu < 0 \)) is determined in the mean field approximation. The equation for the bound state energy \( \epsilon_b \) is obtained and the relationship \( \epsilon_b = 2\mu \) is established. An application of the path integral method to the two-band model is developed and on this basis, the transition from the Fermi to the Bose pattern of elementary excitations in the presence of a two-particle bound state in the system is demonstrated. The expression for the temperature of Bose condensation \( T_k \) is obtained and the contribution of the residual boson interactions is estimated for systems with different dimensions.

1. Introduction

Since the discovery of high-temperature superconductivity (HTSC), scientists have spent a considerable amount of effort clarifying the nature of the phenomenon and explaining the unusual thermodynamic and kinetic properties of metal oxides. However, in spite of significant advances in the field, the mechanisms of HTSC remain unclear. In our opinion, the most interesting of the large number of models and directions, from the theoretical point of view, is the direction connected with accounting for the singularities of the electron energy spectra characteristic of HTSC materials (overlapping energy bands on the Fermi surface, van Hove-Lifshitz singularities, as well as the presence of the "nesting" condition).

In modern-day scientific literature, the model with overlapping energy bands suggested by Moscaenko [1] and Suhl et al. [2] in 1959 is widely used in its different variants. On the basis of that model, we managed to obtain the description of a number of superconducting properties of high-temperature compounds [3-11] in agreement with the experimental data. References to investigations by other authors can be found in the referred articles, as well as in review [12].

The multiband model is popular for a number of reasons, the main ones being the following.

1. In HTSC materials, the existence of the overlapping of two or more energy bands on the Fermi surface is proved and, naturally, this raises the question of the influence of overlapping on the thermodynamic, kinetic, and other properties of superconductors.

2. The model is applicable both for the phonon and nonphonon mechanisms of superconductivity. Even in the most unfavorable case, where repulsion between electrons predominates, HTSC is possible at definite relations between constants of the intra- and interband interactions [7].

3. The model allows the description of the properties of HTSC materials to be closer to the properties observed in experiments.

The important result of these investigations is that accounting for the overlapping of energy bands may lead not only to a quantitative, but also to a qualitative difference from the case of a one-band superconductor. An example of such a difference may be the above-mentioned (item 2) possibility for the appearance of HTSC, the appearance of a step-like dependence of the superconducting transition temperature on the concentration of carriers [5, 11], the positive curvature of the upper critical field near the transition temperature \( T_c \) [6], etc.

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In the articles referred to above, investigations were conducted on the basis of the idea of Cooper pairing. In systems with small carrier concentrations, such as semiconductors or metallic oxides, bound states may arise following a decrease in the carrier concentration, and a transition to a Bose condensate of local pairs with a final binding energy may occur (the Schaffroth scenario [13]). The issue of such transitions in one-band systems was discussed in a number of articles [14–19].

To realize the Schaffroth scenario, a bound two-particle state [20] must exist in the system. The appearance of this state in the presence of attractive interaction depends on the dimension of the system and is realized in the region of small carrier concentrations. As is shown in [15, 18], the change of sign in the chemical potential with a decrease in the carrier concentration corresponds to a transition from the BCS to the Schaffroth scenario. Condensation of local pairs occurs at concentrations of carriers for which \( \mu < 0 \).

This article is devoted to investigations of superconducting ordering in systems with two characteristic features—a small concentration of charge carriers and overlapping energy bands on the Fermi surface. As mentioned above, both of these features are characteristic of superconducting compounds.

Our article is organized as follows. Section 2 is devoted to a self-consistent discussion in the mean field approximation of a system of equations in order parameters \( \Delta_n \) and \( \mu \) at \( T = 0 \) and to revealing the influence of energy band overlapping on these quantities and the carrier concentration at which the system experiences a transition from Cooper pairing (\( \mu > 0 \)) to the Schaffroth (\( \mu < 0 \)) scenario. The equation for the binding energy \( \varepsilon_b \) of a two-particle state is also obtained and the relationship between \( \varepsilon_b \) and \( \mu \) is established. In Sec. 3, the path integral method as applied to the two-band model is developed and, on this basis, the procedure for a transition from the Fermi to Bose elementary excitations is given. The condensation temperature of the Bose system \( T_k \) is also determined. In the last section, the results of the investigation are summarized.

2. Superconductivity at \( T = 0 \) in the mean field approximation

The Hamiltonian describing the two-band system has the form [1, 21]

\[
H = \int dr \sum_{n\sigma} \psi_{n\sigma}^+(r) H_0 n \psi_{n\sigma}(r) - \frac{1}{2} \sum_{nm} V_{nm} \int dr \sum_{\sigma'\sigma} \psi_{n\sigma}^+(r) \psi_{n\sigma'}(r) \psi_{n\sigma'}(r) \psi_{n\sigma}(r) (1 - \delta_{\sigma\sigma'}),
\]

where the first term corresponds to the kinetic energy, the second is responsible for the superconductivity, the band indices \( n, m = 1, 2 \), \( V_{nm} \) are the constants of intra- and interband interaction, \( \psi_{n\sigma}^+(r) \) is the particle creation operator in the band \( n \) with spin \( \sigma \), and \( H_0 n = \frac{\varepsilon_n^2}{2m_n} - \mu_n \). In the mean field approximation and at \( T = 0 \), the order parameters \( \Delta_n \) and the chemical potential \( \mu \) are determined by the self-consistent system of equations [7]

\[
\Delta_n = \sum_{km} V_{nm} \frac{\Delta_m}{2(\varepsilon_n(k) - \mu)^2 + \Delta^2_m},
\]

\[
N = \sum_{kn} \left[ 1 - \frac{\varepsilon_n(k) - \mu}{\sqrt{\varepsilon_n(k) - \mu)^2 + \Delta^2_n}} \right],
\]

where \( \varepsilon_n(k) = \frac{k^2}{2m_n} + \zeta_n \), \( m_n \) is the effective mass, and \( \zeta_n \) is the lowest energy level of the \( n \)th zone.

With a decrease in the carrier concentration at point \( \mu = 0 \), a transition from the BCS state to a Bose condensate of local pairs occurs. We consider the case of a deep Bose condensate of local pairs where the relationships \( \mu < 0 \) (\( \mu' = -\mu > 0 \)) and \( \Delta_n^2/\mu^2 \ll 1 \) hold. In this limit, Eqs. (2) and (3) take the form

\[
\Delta_n = \sum_{m} V_{nm} \Delta_m \xi_m(2\mu'), \quad N = \sum_{kn} \frac{\Delta^2_n}{2(\varepsilon_n(k) + \mu')^2},
\]

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