THE INFLUENCE OF OVERLAPPING ENERGY BANDS ON THE SUPERCONDUCTIVITY OF LOCALIZED BOSE-CONDENSATE PAIRS AT $T = 0$

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A system of equations is obtained for the order parameters $\Delta_{nm}$ and the chemical potential $\mu$ of two-band superconductors with low carrier densities at $T = 0$. The limiting case of $\mu_n < 0$, corresponding to a Bose condensate of localized pairs in both bands, is considered, and all possible inter- and intraband interactions are taken into account. For $\Delta_{nm}^2/\mu_{n,m}^2 \ll 1$, the above system of equations is reduced to a system of linear equations whose analytical solutions are found. The dependences of $\Delta_{nm}$ and $\mu$ on electron-electron interaction constants and carrier densities are analyzed.

1. Introduction

Theoretical investigations of the superconducting properties of rarefied systems on the basis of the one-band model have been conducted by many authors [1-6]. Interest in these systems is connected with describing superconductivity in semiconductors and, after the discovery of high-temperature superconductivity, in metal oxide ceramics.

In addition to accounting for such peculiarities as reduced carrier concentrations and reduced dimensionality, the important characteristic of metal oxide ceramics is the overlapping of energy bands on the Fermi surface.

The presence of such overlapping was confirmed by a large number of band model calculations (e.g., [7, 8]). The multi-band model-based theory of superconductivity [9, 10] allows the properties of metal oxide ceramics to be described under the assumption of both phonon and nonphonon mechanisms of superconductivity, as well as explaining many of the anomalies in the superconducting characteristics observed in experiments. In [11-16], two- and three-band models were used to describe the properties of superconductors for which the relationship $\mu \gg T_c$ was correct ($\mu$ is the chemical potential and $T_c$ is the superconducting transition temperature). In a number of cases, this relationship is also valid for high-temperature superconductors. The existence of such a relationship between $\mu$ and $T_c$ allows the approximation diagonal, with respect to the band indices, to be used in calculations [17-19] and, thus, permits the nondiagonal order parameters $\Delta_{12}$ and $\Delta_{21}$ to be neglected.

In systems with low carrier concentrations, however, the relationship $\mu \gg T_c$ does not hold. As a result, the necessity of constructing a theory of superconductivity in two-band systems without imposing restrictions on the Fermi energy arises. Such investigations were performed in [20] for the nonphonon mechanism of superconductivity and in [21] for the phonon mechanism. In these papers, the carrier concentration dependences of $T_c$ and heat capacity jumps were determined, and the influence of energy band overlapping on the bend in the temperature dependence of the chemical potential at $T = T_c$ was investigated. In the mean field approximation, this theory is valid for any carrier concentration, including $\mu \sim T_c$. However, the mean field approximation can become insufficient when very low carrier concentrations are considered. The necessity then appears to account for order parameter fluctuations in the vicinity of the superconducting...
transition temperature. Therefore, to perform numerical calculations, we limit ourselves to the condition $T_c/\mu > 10^{-1}$. Thus, in [20, 21] we assumed the conventional superconductivity corresponding to Cooper pairing.

As was shown in [2, 4], as the carrier concentration decreases, the transition from the Bardeen-Cooper-Schrieffer (BCS) state to a Bose condensate of localized pairs occurs. In this case, it is assumed that condensation occurs for carrier concentrations such that $\mu < 0$.

In our opinion, continuing [20, 21], the investigation of the superconducting properties of two-band systems for low carrier concentrations ($\mu_n < 0$) and $T = 0$ is of interest. In the given case, $\mu_n < 0$ corresponds to a Bose condensate in both bands ($n = 1, 2$), and the condition $T = 0$ allows one to apply the mean field approximation to the description of systems with low carrier concentrations. This is because the description of the Bose condensate of localized pairs qualitatively coincides with the mean field description at $T = 0$ [22]. In this paper, analytical expressions for the order parameters $\Delta_{nm}$ and the chemical potential $\mu$ will be obtained in order to make conclusions about their concentration dependences and the role of the overlapping energy bands.

This article is organized as follows. In Section 2, the Hamiltonian of the system is presented, the system of equations for the order parameters $\Delta_{nm}$ and the chemical potential $\mu$ at $T = 0$ is written, and the passage to the limit $\mu_n < 0$ and $(\Delta_{nm}/\mu_{nm})^2 \ll 1$ is performed. In Section 3, analytical solutions for $\Delta_{nm}$ and $\mu$ are found, taking into account all kinds of electron–electron interactions. Section 4 is devoted to considering simpler limiting cases. In Section 5, the difference between the thermodynamic potentials for the trivial $\Delta_{nm} \neq 0$ and nontrivial $\Delta_{nm} = 0$ solutions is calculated. Section 6 contains our conclusions.

In this article, we do not refer to the investigations of superconducting two-band systems by other authors because they do not impinge on ours. These references can be found in our previous works [14–16, 20, 21], as well as in [23].

### 2. The Hamiltonian of the system and basic equations for $T = 0$

The two-band superconducting system with an arbitrary carrier concentration is described by the Hamiltonian [20, 21]

$$H = \sum_{nk\sigma} [\varepsilon_n(k) - \mu] a_{nk\sigma}^+ a_{nk\sigma} - \frac{1}{V} \sum_{m_1...m_4} V_{m_1m_2m_3m_4} a_{m_1,k\uparrow}^+ a_{m_2,-k\downarrow} a_{m_3,-k'\downarrow} a_{m_4,k'\uparrow},$$

where $a_{nk\sigma}^+$, $a_{nk\sigma}$ are, respectively, the creation and annihilation operators of an electron with spin $\sigma$ and quasiwave vector $k$ in the $n$th band, and $V_{m_1m_2m_3m_4}$ are the constants of inter- and intraband interactions. Expression (1) is the generalization of the BCS-Bogoliubov Hamiltonian to the two-band case. For $m_1 = m_2$ and $m_3 = m_4$, Hamiltonian (1) transforms into the corresponding expression of the Moskalenko model [9] considering only interband pairing and the transitions of Cooper pairs as a whole from one band to another.

Using Hamiltonian (1) and the Green functions method [24], the system of equations for the order parameters $\Delta_{np}$ ($n, p = 1, 2$) for $T = 0$ [21, 22] is obtained

$$\Delta_{np} = \frac{1}{V} \sum_{k,l,r} V_{np}^{ll} \Delta_{ll} \left\{ \left[ \frac{\xi_l^2 - \xi_r^2 + 2\Delta_{12}\Delta_{21}(1 + 1/z_{ir})}{d} + 1 \right] \frac{1}{E_1} - \left[ \frac{\xi_l^2 - \xi_r^2 + 2\Delta_{12}\Delta_{21}(1 - 1/z_{ir})}{d} - 1 \right] \frac{1}{E_2} \right\} + \frac{1}{V} \sum_{k} \left( V_{np}^{12} \Delta_{21} + V_{np}^{21} \Delta_{12} \right) \cdot \Gamma,$$

where

$$\Gamma = \left[ \frac{\xi_l^2 - \xi_r^2 + 2\Delta_{12}\Delta_{21}(1 + 1/z_{ir})}{d} + 1 \right] \frac{1}{E_1} - \left[ \frac{\xi_l^2 - \xi_r^2 + 2\Delta_{12}\Delta_{21}(1 - 1/z_{ir})}{d} - 1 \right] \frac{1}{E_2} + \frac{1}{V} \sum_{k} \left( V_{np}^{12} \Delta_{21} + V_{np}^{21} \Delta_{12} \right) \cdot \Gamma,$$