NONLINEAR MODE-INTERACTION IN THE MACROECONOMY

Erik MOSEKILDE1, Erik REIMER LARSEN2, John D. STERMAN3 and Jesper SKOVHUS THOMSEN1

1 Physics Laboratory III, Technical University of Denmark, DK-2800 Lyngby, Denmark
2 Institute of Economics, Copenhagen Business School, DK-1366 Copenhagen K, Denmark
3 Sloan School of Management, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract

A central problem for a dynamic formulation of macroeconomic theory is how to explain the occurrence of different, relatively well-defined economic modes such as the short term business cycle, the construction (or Kuznets) cycle, and the economic long wave (or Kondratiev cycle). Equally important is a description of the various phenomena that can arise through interaction between these cycles. Modern nonlinear theory suggests that different cyclical modes may be entrained through the process of mode-locking, where the periods of the interacting modes adjust to one another, so as to attain a rational ratio. This type of interaction is well documented in physical and biological systems. However, despite the importance of the problem and abundant evidence for nonlinearity in the economy, modern concepts of nonlinear mode-interaction have not yet been applied to the problem of entrainment between economic cycles. We show how mode-locking and other highly nonlinear dynamic phenomena arise in a model of the economic long wave. The behavior of the model is mapped as a function of the frequency and amplitude of an external forcing, producing both a devil’s staircase and a detailed Arnol’d tongue diagram. Two different routes to chaos are identified. The Lyapunov exponents are calculated, allowing the strength of the chaos to be assessed, and the fractal nature of the basins of attraction for two simultaneously existing periodic solutions is illustrated. The paper concludes with a discussion of the implications for economic theory.

Keywords: Nonlinear economics, mode-locking, devil’s staircase, period doubling, chaos, fractal basin boundaries.

1. Introduction

A central problem for macroeconomists and economic historians is how to explain the different types of economic fluctuations and the large historical variation in the size of business downturns experienced by industrialized nations. Why are some recessions mild and short while others, such as the great depressions, are long and deep? Economists have identified a number of distinct oscillatory modes operating simultaneously in the macroeconomy, including the short-term business cycle of 3 to 5 years [1–3], the 15 to 25 year construction or Kuznets cycle [4–6], and the Kondratiev or economic long wave [7–10].

© J.C. Baltzer AG, Scientific Publishing Company
Some authors, including Long [5] have sought to explain periods of severe depression such as the 1930's as the simultaneous downturn of several cycles. If the economy were completely linear, its total behaviour would be the superposition of the separate modes, and the coincidence of downturns in each cycle would indeed produce a large depression. However, such an explanation leaves unanswered a number of basic questions. If the economy is linear, why do the individual sectors of the economy adjust their changes in employment and production so as to produce a coherent motion of the economy as a whole? And why, as suggested by Schumpeter [11], do the periods of the most prominent cycles appear to be multiples of one another?

The purpose of the present paper is to apply modern concepts of nonlinear mode-interaction [12, 13] to the problem of macroeconomic entrainment. We begin with a few empirical examples of nonlinear interaction among modes in various industries. We then introduce a model of the economic long wave developed at the Sloan School of Management [14], and show how it produces an endogeneous variation in aggregate production with a period of approximately 50 years. Next, we demonstrate how the simple model can exhibit mode-locking and other highly nonlinear dynamic phenomena when subjected to an external sinusoidal forcing. This forcing is meant to represent perturbations of the economic long wave by shorter term economic fluctuations. To understand how and to which degree such perturbations can influence the dynamics, we map the behaviour of the model as a function of the frequency and amplitude of the sinusoidal forcing. Two different routes to chaos are identified, the Lyapunov exponents are calculated, and the fractal nature of the basins of attraction for two simultaneously existing periodic solutions is illustrated. Finally, implications for dynamic analysis of economic fluctuations are discussed.

2. Evidence for nonlinear interactions in the economy

If the economy were linear, the different modes could evolve independently of one another, and their underlying causes could be studied separately. But there is strong empirical and theoretical evidence to suggest that nonlinearity plays a critical role in the interaction of the different modes and in determining the overall behavior.

As an example, fig. 1(a) shows the variation in oil-tanker spot rates from 1950 through 1988. Spot rates are characterized by a series of sharp peaks and deep valleys occurring at 3 to 5 year intervals, separated by periods of 10 to 15 years in which rates are low and relatively constant. During the peaks, which often last for only a few months, rates of more than 400 are attained while during the depression period rates are as low as 40. A variation of this magnitude surely brings us outside the linear regime. The pattern of alternating calm and wild swings is explainable in terms of the nonlinear interaction of the tanker construction cycle with a characteristic