INVESTIGATION OF THE ENERGY OPERATOR SPECTRUM
OF THE TWO-MAGNON SYSTEM IN A ONE-DIMENSIONAL
SPIN \( s = 1 \) NON-HEISENBERG FERROMAGNET WITH NEAREST
AND SECOND-NEAREST NEIGHBOR INTERACTIONS

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We consider the two-magnon system in a one-dimensional non-Heisenberg ferromagnet of spin \( s = 1 \) with
nearest and second-nearest neighbor interactions. We demonstrate that at most two two-magnon bound states
can exist in the system for \( \Lambda = \pi \) and \( \Lambda = 0 \), and at most six such states for \( \Lambda = \frac{\pi}{2} \).

1. Introduction

The two-magnon system in a one-dimensional Heisenberg ferromagnet of spin \( s = \frac{1}{2} \) with interactions
up to the second-nearest neighbors was considered in [1]. As was shown in the one-dimensional case,
the Hamiltonian of the system has a unique bound state with an energy below the continuous spectrum
of \( \hat{H}_{2\Lambda} \) if the center of mass momentum of the bound pairs takes the value \( \Lambda = \pi \). This is true, provided
the parameter \( \alpha \) (the ratio of the second-neighbor bilinear exchange interaction parameter to the nearest-
neighbor bilinear exchange interaction parameter) takes the values \( 0 \leq \alpha < \frac{1}{2} \). For \( \alpha \geq \frac{1}{2} \), this bound state
disappears. In the case of \( \Lambda = 0 \), there are no bound states in the system. For \( \Lambda = \frac{\pi}{2} \), the system has
a unique two-magnon bound state (TMBS) with an energy below the continuous spectrum of \( \hat{H}_{2\Lambda} \) for all
values of \( \alpha \). For the other values of \( \Lambda, -\pi < \Lambda < \pi \), there exists a value of \( \alpha \), denoted as \( \alpha_{cr} \), such that
for \( 0 \leq \alpha < \alpha_{cr} \) the system has a unique TMBS with an energy below the continuous spectrum of \( \hat{H}_{2\Lambda} \).
If, on the other hand, \( \alpha \geq \alpha_{cr} \), then this bound state disappears. Let us note that \( \alpha_{cr} \) depends on the
parameters \( \Lambda \) and \( J \).

This analysis was carried over in [2] to the case of the third-neighbor interaction at \( \Lambda = \pi \). It was shown
that there exist three subsets \( G_j, j = 1, 2, 3 \), of the unit square \( Q = \{(\alpha;\beta) : 0 \leq \alpha \leq 1, 0 \leq \beta \leq 1\} \) such
that a unique TMBS with an energy below the continuous spectrum of \( \hat{H}_{2\Lambda} \) exists for \((\alpha;\beta) \in \bigcup_{j=1}^{3} G_j \).
These subsets are:

- \( G_1 = \{ (\alpha;\beta) \in Q : \alpha > \frac{1}{3}, \beta > \frac{\alpha(2\alpha - 1)}{3\alpha - 1} \} \),
- \( G_2 = \{ (\alpha;\beta) \in Q : \alpha < \frac{1}{3}, \beta < \frac{\alpha(2\alpha - 1)}{3\alpha - 1} \} \),
- \( G_3 = \{ (\alpha;\beta) \in Q : \alpha = \frac{1}{3}, 0 \leq \beta \leq 1 \} \).

For the other values of \((\alpha;\beta) \in Q\), this bound state disappears, its energy being absorbed by the continuous
spectrum.

In the present paper, we take a one-dimensional integral lattice and consider the two-magnon spin \( s = 1 \)
system with nearest and second-nearest-neighbor interactions in a non-Heisenberg ferromagnet. We study
the TMBS of this system for \( \Lambda \) equal to \( \pi, \frac{\pi}{2} \) and \( 0 \). It is shown that the system with \( \Lambda = \pi \) and \( \Lambda = 0 \) can

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have at most two TMBS whose energies lie outside the continuous spectrum of $\widetilde{H}_{2\Lambda}$, and at most six such TMBS for $\Lambda = \frac{\pi}{2}$.

2. The Hamiltonian and its invariant subspace

The Hamiltonian of the system under consideration reads

$$ H = -J \sum_{m, r, k=1}^{2} \alpha_k (S_m \cdot S_{m+kr}) - J_1 \sum_{m, r, k=1}^{2} \left[ \sum_{k=1}^{2} \alpha_k (S_m \cdot S_{m+kr}) \right]^2, \quad (1) $$

where $J > 0$ and $J_1 > 0$ are the bilinear and biquadratic exchange interaction parameters, respectively, and $\tau$ denotes summation over $\pm 1$; $\tau = \pm 1$, $\alpha_1 = 1$ and $\alpha_2 = \alpha$, $0 \leq \alpha \leq 1$.

Let $Z$ be a one-dimensional integral lattice. Denote by $S_m = (S^x_m, S^y_m, S^z_m)$ the atomic spin-$s = 1$ operator at node $m$ and let $S_m^\pm = S_m^x \pm i S_m^y$. Denote by $\varphi_0$ the vector, which is called the vacuum, and let it be uniquely defined by the conditions $S_m^+ \varphi_0 = 0$, $S_m^- \varphi_0 = \varphi_0$ and $||\varphi_0|| = 1$. The vectors $S_m^+ S_n^- \varphi_0$ describe the state in which two magnons of spin $s = 1$ are sitting at nodes $m$ and $n$ and make up an orthonormal system. The space spanned by these vectors is denoted as $\mathcal{H}_2$, which is Euclidean with respect to a natural scalar product.

Operator (1) acts in the space of magnon states $\mathcal{H}_{\text{mag}}$, where $\mathcal{H}_{\text{mag}} = \bigotimes_k \mathbb{C}^3_k$, with $\mathbb{C}^3_k \equiv \mathbb{C}^3$ for arbitrary $k \in Z$, is an infinite tensor product of the three-dimensional Euclidean spaces $\mathbb{C}^3$.

Observe that the restriction $H_2$ of $H$ onto the space $\mathcal{H}_2$ is a bounded self-adjoint operator. The construction of $\mathcal{H}_2$ and the explicit form of $H$ lead to the following proposition.

Proposition 1. The space $\mathcal{H}_2$ is invariant under the action of $H$. The operator $H_2 = H/\mathcal{H}_2$ is bounded and self-adjoint, and gives rise to a bounded self-adjoint operator $\widetilde{H}_2$ that acts in $l_2(\mathbb{Z} \times \mathbb{Z})$ as

$$ (\widetilde{H}_2 f)(p; q) = -J \sum_{\tau, p, q} \sum_{k=1}^{2} \left[ \alpha_k (\delta_{p,q+kr} + \delta_{p+kr,q} - 4) f(p; q) - \frac{1}{2} \alpha_k \delta_{p-kr,q} f(p - k\tau; q) - \frac{1}{2} \alpha_k \delta_{p+kr,q} f(p + k\tau; q) - \frac{1}{2} \alpha_k \delta_{p,q-kr} f(p; q - k\tau) - \frac{1}{2} \alpha_k \delta_{p,q+kr} f(p; q + k\tau) + \alpha_k f(p - k\tau; q) + \alpha_k f(p; q - k\tau) + \alpha_k f(p + k\tau; q) + \alpha_k f(p; q + k\tau) \right] - J_1 \sum_{\tau, p, q} \sum_{k=1}^{2} \left[ \alpha_k (2\delta_{p,q} - 2\delta_{p+kr,q} - 2\delta_{p,q+kr} + 8) f(p; q) + \alpha_k \delta_{p,q+kr} f(p - k\tau; q + k\tau) + \alpha_k \delta_{p+kr,q} f(p + k\tau; q - k\tau) - 2\alpha_k f(p - k\tau; q) + \alpha_k \delta_{p,q} f(p - k\tau; q + k\tau) + \alpha_k \delta_{p,q+kr} f(p - k\tau; q) - \alpha_k \delta_{p,q} f(p - k\tau; q) - 2\alpha_k f(p + k\tau; q) - 2\alpha_k f(p; q - k\tau) + \frac{1}{2} \alpha_k \delta_{p,q+kr} f(p + k\tau; q) + \frac{1}{2} \alpha_k \delta_{p,q} f(p + k\tau; q) - \alpha_k \delta_{p,q+kr} f(p + k\tau; q) - \alpha_k \delta_{p,q} f(p + k\tau; q) - \alpha_k \delta_{p,q} f(p; q + k\tau) + \frac{1}{2} \alpha_k \delta_{p,q+kr} f(p; q + k\tau) + \frac{1}{2} \alpha_k \delta_{p,q} f(p; q + k\tau) - \alpha_k \delta_{p,q+kr} f(p; q + k\tau) - \alpha_k \delta_{p,q} f(p; q + k\tau) \right], $$

where $\delta_{k,j}$ is the Kronecker symbol. As to the operator $H_2$ itself, its action on the vector $\Psi \in \mathcal{H}_2$ is given by

$$ H_2 \Psi = \sum_{p, q} (\widetilde{H}_2 f)(p; q) S_p^- S_q^- \varphi_0. \quad (2) $$