Auf alle Fälle stellen grosse Ruderausschläge bei hohen Machschen Zahlen, zumindest am Höhenleitwerk, eine Gefahr dar. Strömungen mit Verdichtungsstössen und Grenzschichtablösung sind der Berechnung nicht zugänglich und sind durch sehr kleine Ursachen beeinflussbar. Das Absinken der Ruderwirkung kann bald mehr und bald weniger stark und auch mehr oder weniger steil sein, und man kann sich im Einzelfall auf Messungen nicht verlassen. Man vermeidet diese Gefahr am einfachsten, indem man oberhalb einer gewissen Fluggeschwindigkeit an Stelle grosser Ruderausschläge entsprechende Flossentrimmungen verwendet.

Summary

It is called attention to the danger for airplanes resulting from the behaviour of the rudder efficiency at overcritical Mach numbers. Windtunnel tests show that in many cases at transsonic Mach numbers a sharp decrease of the rudder efficiency is possible. Author shows that as a consequence of this behaviour under certain conditions forces may be produced which are ample to destroy the aircraft.

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Growth of Boundary Layer on a Rotating Sphere

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1. Introduction

Howarth\(^1\) has investigated the problem of the flow engendered by a sphere rotating uniformly about a diameter in otherwise undisturbed fluid. From the physical considerations we should expect an inflow at the poles and an outflow in the equatorial plane. The flow should be symmetrical about the equatorial plane, and the sphere may be considered to be made up of two hemispheres joined smoothly at the equator. Howarth finds that the boundary layers originate at the poles on the two hemispheres and develop towards the equator where they impinge on each other. Howarth's solutions are not valid near the equator. They do not give any information about the flow in the equatorial plane. He concludes on the basis of his solutions that inflow will occur over a large part of the surface, the outflow necessary to maintain continuity will be confined to the vicinity of the equatorial plane. His solutions give an inflow at the poles, but they do not give any outflow in the equatorial plane because they cease to be valid near the equator.

In the present note we have discussed the growth of motion, in the earlier stages of its development, caused by a sphere which at the time \(t = 0\) is suddenly made to rotate with a constant angular spin \(\Omega\) about a diameter in fluid otherwise undisturbed. The solutions have a serious limitation in that they give initial motion only. They give no information regarding the time after which the steady state is established.

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1) Indian Institute of Technology, Kharagpur, India.
2) L. Howarth, Phil. Mag. 42, 1308-1315 (1951).
2. Equations of motion

We shall use spherical polar coordinates $r, \theta, \varphi$ with $r$ measured radially outwards from the centre of the sphere, $\theta$ measured from the axis of rotation, and $\varphi$ the azimuth. If $w, u, v$ represent the components of velocity in the directions $r, \theta, \varphi$ increasing the boundary layer equations can be deduced from Howarth\textsuperscript{1}) by placing $K_1 = 0, K_2 (1/a) \cot \theta$, where $a$ is the radius of the sphere. They are:

\begin{align*}
\frac{\partial u}{\partial t} + \frac{u}{a} \frac{\partial u}{\partial \theta} + w \frac{\partial u}{\partial r} - \frac{v^2}{a} \cot \theta &= v \frac{\partial^2 u}{\partial r^2} , \\
\frac{\partial v}{\partial t} + \frac{u}{a} \frac{\partial v}{\partial \theta} + w \frac{\partial v}{\partial r} + \frac{uv}{a} \cot \theta &= v \frac{\partial^2 v}{\partial r^2} , \\
\frac{1}{a} \frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial r} + \frac{u}{a} \cot \theta &= 0 .
\end{align*}

We now set

\begin{align*}
u &= \Omega^2 a \sin \theta \cos \theta \, t \, f(\eta) , \\
v &= \Omega a \sin \theta \, g(\eta) , \\
w &= -\Omega^2 2 \nu^{1/2} t^{3/2} (3 \cos^2 \theta - 1) \, h(\eta) ,
\end{align*}

where

$$\eta = \frac{r - a}{2 \sqrt{(v t)^{1/2}}} .$$

During the early stages of motion when $t$ is small (or in boundary layer theory terminology: when the thickness of the boundary layer is small), we may neglect the terms in the equations of motion containing higher powers of $t$. Therefore, omitting terms of order $t^2$ in the equations of motion, we get to a first order of approximation the following equations:

\begin{align*}
f'' + 2 \eta \, f' - 4 f &= -4 \eta^2 , \\
g'' + 2 \eta \, g' &= 0 , \\
f &= h' ,
\end{align*}

where a dash denotes differentiation with respect to $\eta$.

These equations are the same as the equations in the problem of boundary layer growth over an infinite rotating disk (Nigam\textsuperscript{2}).

3. Solutions of the equations

The solutions of these equations satisfying the boundary conditions $u = 0$, $v = \Omega a \sin \theta$, $w = 0$ on the sphere $r = a$; and $u = v = 0$ at $r = x$, are

\begin{align*}
g &= [1 - \text{erf} \, \eta] = \text{erfc} \, \eta , \\
f &= \frac{2}{\pi} \left[(1 + 2 \eta^2) \, \text{erfc} \, \eta - 2 \pi^{-1/2} \eta \, e^{-\eta^2} \right] - 2 \left(\pi^{-1/2} \, e^{-\eta^2} - \eta \, \text{erfc} \, \eta \right)^2 , \\
h &= \frac{2}{3 \pi} \left[(3 \eta + 2 \eta^3) \, \text{erfc} \, \eta - 2 \pi^{-1/2} (1 + \eta^2) \, e^{-\eta^2} \right] - \frac{2}{3} \eta \left(\pi^{-1/2} \, e^{-\eta^2} - \eta \, \text{erfc} \, \eta \right)^2 \\
&\quad - \frac{2}{3 \pi^{1/2}} \, e^{-\eta^2} \, \text{erfc} \, \eta + \frac{2 \sqrt{2}}{3 \pi^{1/2}} \, \text{erfc} \left(\sqrt{2} \eta\right) + \frac{2}{3 \pi^{1/2}} \left(\frac{2}{\pi} - 2^{1/2} + 1 \right) .
\end{align*}

\textsuperscript{1}) L. Howarth, Phil. Mag. 42, 239–243 (1951).

\textsuperscript{2}) S.D. Nigam, Quart. Amer. Math. 9, 89–91 (1951).