A Systematic Comparison between Born Approximation and Distorted Wave Born Approximation for Inelastic Electron Scattering*

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Received December 16, 1967

The correction factors for Coulomb effects in inelastic electron scattering are given for electric quadrupole transitions. The cross sections in Born approximation and distorted wave Born approximation are calculated in the liquid drop model for electron energies between 20 and 80 MeV and nuclei up to \( Z = 26 \).

1. Introduction

It is well known, that the first Born approximation (BA) for electroexcitation is valid only for scattering on light nuclei. Even in these light nuclei, the Coulomb effects may rise to corrections larger than the experimental errors if the electron energy is small and/or the momentum transfer large. Instead, the cross section has to be calculated using the electron wave functions in the static Coulomb potential of the nucleus (DWBA)\(^1-^4\). The experimental results should be corrected by the ratio of the cross sections in BA and DWBA, calculated in a specific nuclear model. Of course this model should be consistent with the experimental data. At low momentum transfer this is easily achieved by iteration.

In section 2 we briefly review the formulae for the cross sections in BA and DWBA. The nuclear model will be specified in section 3. Finally, in section 4, we give the correction factors together with a discussion of the accuracy and some conclusions.

* Work supported by the Deutsche Forschungsgemeinschaft and in part carried out under the auspices of the Center for Advanced Studies of the University of Virginia.

2. The Cross Section for Electro-Excitation

Neglecting the rest mass of the electron, the excitation energy of the nucleus and the transverse parts of the interaction, the differential cross section in DWBA is

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{DWBA}} = \left( \frac{V}{2\pi} \right)^2 \frac{E^2}{2} \sum_{m,m',\mu} \left| \langle f | H_{\text{int}} | i \rangle \right|^2. \quad (1)$$

In the following we will assume that the spin of the nucleus is 0 in the initial state and \( \lambda \) in the final state.

$$\langle f | H_{\text{int}} | i \rangle = \left( \frac{2\pi}{V} \right) \frac{\pi}{E^2} \sum_{\kappa,\kappa'} e^{i(\delta_{\kappa} + \delta_{\kappa'})} \cdot l^{(l'-l)} \cdot (-)^{m' + \frac{1}{2}} (2j + 1)\cdot (2j' + 1) \sqrt{(2\lambda + 1)(2l + 1)} \left( \begin{array}{ccc} l & \frac{1}{2} & j \\ 0 & -m & j' \end{array} \right) \left( \begin{array}{ccc} l' & \frac{1}{2} & j \\ m - m' & -m' & -\mu + m \end{array} \right) \cdot \left( \begin{array}{ccc} j' & \lambda & j \\ \mu - m & -\mu & m \end{array} \right) \cdot \frac{1}{2} \left( 1 + (-)^{l + l'} \right) R_{\kappa',\kappa}^{(\lambda,\mu)} Y_{l', \mu'} Y_{l, -\mu + m - m'.} \cdot \hat{p}_f. \quad (2)$$

For details of the notation see ref.\(^4\).

The radial integrals are defined by

$$R_{\kappa',\kappa}^{(\lambda,\mu)} = \int_0^\infty dr e (f_{\kappa'} f_{\kappa} + g_{\kappa'} g_{\kappa}) J_{\lambda,\mu}(r). \quad (3)$$

The transition potential \( J_{\lambda,\mu} \) is given by the matrix elements of the transition density operator \( \rho_N(r) \) between the nuclear wave functions in the initial and final states.

$$J_{\lambda,\mu}(r_e) = -4\pi \alpha (2\lambda + 1)^{-1} r_e^2 \int_0^\infty d\tau Y_{\lambda,\mu}(\Omega) \frac{r_e^2}{r_e^2 \tau^2} \langle f | \rho_N(r) | i \rangle. \quad (4)$$

The cross section in Born approximation is

$$(d\sigma/d\Omega)_{\lambda}^{\text{BA}} = \sigma_M \cdot F^2_\lambda \quad (5)$$

where \( \sigma_M \) is the Mott cross section

$$\sigma_M = 4(\alpha Z)^2 E^2 \cos^2 \Theta/2 \cdot q^{-4} \quad (6)$$

and \( q \) the momentum transfer

$$q = 2E \sin \Theta/2. \quad (7)$$

The form factor is given by

$$F^2_\lambda = 4\pi \left[ \langle \lambda || M(C,\lambda) || 0 \rangle \right]^2, \quad (8)$$

with

$$M(C,\lambda) = 1/Z \int d\tau \rho_N(r) j_{\lambda}(q r) Y_{\lambda,\mu}(\hat{r}). \quad (9)$$