Effective Potential for Scalar QED in (2 + 1) Dimensions

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We calculate the renormalized one loop approximation to the effective potential for scalar electrodynamics in (2 + 1) dimensions. We study its gauge dependence and show that it satisfies the renormalization group equation since it is independent on any renormalization scale. This relies on the fact that we do not need to improve the effective potential in (2 + 1) dimensions.

1. INTRODUCTION

In general the study of lower dimensional situations is motivated by the fact that in the simpler setting we can learn useful things which can be applied to four-dimensional problems. Moreover, there are possible physical applications: The high-temperature behavior of four-dimensional field theories is governed by their three-dimensional analysis (Gross et al., 1981; Dese et al., 1982). Interesting condensed matter phenomena such as the quantum Hall effect and high-\( T_c \) superconductivity appear to involve planar gauge-theoretic dynamics. Introducing anyons in (2 + 1)-dimensional physics created a new point of contact between solid-state physics and particle physics. Anyonic superconductivity presents a candidate for the explanation of the superconducting properties observed in certain materials at high temperature.

The effective potential for a field theory as introduced by Euler, Heisenberg, and Schwinger is very useful in the study of spontaneous symmetry breaking. Unfortunately, an exact computation of the effective potential involves an infinite number of Feynman diagrams (Coleman and Wienberg, 1973). This is a difficult task, especially when several interactions are present.
So we are only able to calculate the effective potential approximated to a few-loop approximation.

The expansion of the effective potential in four dimensions contains a renormalization scale which is arbitrary. The effect of its change can be absorbed into the changes in the coupling constant and the field. This problem arises from the fact that the effective potential does not satisfy the renormalization group equation. The renormalization group equation for the effective potential states that

$$\frac{dV}{dM} = 0$$

Then

$$\left[ M \frac{\partial}{\partial M} + \beta(g_i) \frac{\partial}{\partial g_i} - \gamma \phi \frac{\partial}{\partial \phi} \right] V = 0$$

where $\beta = M \frac{dg_i}{dM}$ and $\gamma$ is the anomalous dimension, $\gamma = -\frac{d\phi}{dM}$. Many procedures to improve the effective potential so as to satisfy the renormalization group equation have been suggested (McKeon, 1984; Kastening, 1991; Bando et al., 1992).

In this note, we calculate the renormalized one-loop approximation to the effective potential for scalar electrodynamics in $(2 + 1)$ dimensions. This expression of the effective potential is required for studying the spontaneous symmetry breaking in $(2 + 1)$ dimensions.

We study the gauge dependence of the effective potential; this dependence presents a difficulty in using the effective potential, since it may create false minima. We show explicitly this gauge dependence of the effective potential in $(2 + 1)$ dimensions.

We note that the effective potential is independent on any renormalization scale, in contrast with the effective potential in four dimensions, where $V(\phi) \approx \phi^4 [\ln(\phi^2/M^2) - \frac{1}{2}]$ and $M$ is the renormalization scale. So in $(2 + 1)$ dimensions we do not need to introduce an improvement to the effective potential, since it already satisfies the renormalization group equation.

2. EFFECTIVE POTENTIAL FOR SCALAR QED IN 2 + 1 DIMENSIONS

In scalar massless quantum electrodynamics,

$$S(\phi, A) = \int d^3x \left[ -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + |(\partial_\mu + ieA_\mu)\phi|^2 - \frac{\lambda}{4} (|\phi|^2)^2 \right]$$