Elementary Particle States Based on the Clifford Algebra \( C_7 \)

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The lepton isodoublet \((e^-, \nu_e)\), the “bare” nucleon isodoublet \((n, p)\), and their antiparticles are shown to constitute a basis of the irreducible representation of the Clifford algebra \( C_7 \). The excited states of these doublets, i.e., \((\mu^-, \nu_\mu)\), \((\tau^-, \nu_\tau)\), ..., and \((s^0, c^+), (b^0, t^+)\) are generated by the products \((e^-, \nu_e) \otimes a\) and \((n, p) \otimes a\), where \(a \equiv 2^{-1/2}(e^- e^+ + \nu_e \bar{\nu}_e)\) has the same quantum numbers as the photon state. The bare baryons \(s, c, b, t\) carry the strangeness, charm, bottom, and top quantum numbers. These lepton and bare baryon states are in one-to-one correspondence with the integrally charged colored Han-Nambu quarks, and generate all the observed \(su(3)\) and \(su(4)\) hadron multiplets.

1. INTRODUCTION

Clifford algebras \(C_n\) (Clifford, 1878; Brauer and Weyl, 1935; Riesz, 1958; Hestenes, 1966; Kahan, 1966) play an important role in physics, as evidenced by the Pauli spin algebra \(C_2\) and the Dirac algebra \(C_4\). In increasing order, Eddington (1946) used \(C_4\) in his fundamental theory, Barut and Haugen (1973) used \(C_6\) in their formulation of conformally invariant massive spinor equations and the \(e - \mu\) system, and Basri and Horwitz (1975) used \(C_7\) to describe the hadronic mass spectrum. More recently, Casalbuoni and Gatto (1980) used higher-order Clifford algebras in a unified description of quarks and leptons. They use a gauge theory and orthogonal groups such as \(o(13, 1)\), that are generated by higher-order Clifford algebras.
The main goal of this paper is to show how $C_7$ and its tensor products can be used to generate all observed particle multiplets; dynamics is outside the scope of this paper. In particular, we use $C_7$, its tensor products, and an orbital $o(4,2)$ algebra, to generate two sequences of isodoublets; one is the lepton sequence $(e^-, \nu_e)$, $(\mu^-, \nu_\mu)$, $(\tau^-, \nu_\tau)$, ..., and the other is the baryon sequence $(n, p)$, $(s, c)$, $(b, t)$, ..., where $n, p$ are the "bare" nucleons, and $s, c, b, t$, are the "bare" hyperons carrying the "strangeness," "charm," "bottom," and "top" quantum numbers (QN), respectively. All particle states are obtained as tensor products of these states.

The essential properties of Clifford algebras and their physical identifications are outlined in Section 2. These principles are applied to $C_7$ in Section 3, and to the Dirac subalgebra in Section 4. The associated orbital algebra is presented in Section 5. Then the eigenstates of the complete algebra are given in Section 6. An irreducible representation (IR) of the isospin algebra, commuting with the Dirac algebra, is derived in Section 7. Excited lepton and bare baryon states are constructed in Section 8, meson states in Section 9, and baryon states in Section 10. It is shown in Section 10 that the lepton and "bare" baryon states play the role of the integrally charged colored Han–Nambu quarks. Regge trajectories are briefly discussed in Section 11, and the basic results are summarized in Section 12.

2. CLIFFORD ALGEBRAS

We outline here the basic facts about Clifford algebras necessary for this work. A (complex) Clifford algebra $C_n$ is generated by the identity $e$, and $n$ elements $e_1, \ldots, e_n$ satisfying the relations

\[ e_A^2 = -e, \quad e_A e_B = -e_B e_A \quad \text{for } A \neq B \]  

The remaining elements of $C_n$ are obtained from all possible products of $e_A$. The number of elements that are a product of $k$ different $e_A$ is \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \), and the total number of elements of $C_n$ is \( \Sigma_{k=0}^{n} \binom{n}{k} = 2^n \).

For even $n$, there is no element besides the identity $e$ that commutes with all the elements of $C_n$, i.e., the center of $C_n$ consists of $e$ only. However, for odd $n$, the center consists of $e$ and the element $e_1 e_2 \cdots e_n$.

If we set

\[ f_n \equiv ie_1 \cdots e_n \quad \text{for } n = 1, 2, 5, 6 \]  
\[ f_n \equiv e_1 \cdots e_n \quad \text{for } n = 3, 4, 7 \]