No More Spacetime Singularities?†

V. Alan Kostelecký¹ and Malcolm Perry²

We discuss the possibility that the issue of spacetime singularities in general relativity is solved by their stringy extensions.

General relativity is a classical theory of gravitation and spacetime. Perhaps its most spectacular success is its application to the universe as a whole and the related description of big-bang cosmology from an era of about $10^{-35}$ seconds until the present. Nonetheless, there are two major difficulties with the theory. The first is a problem afflicting any classical theory, namely, whether it can be derived as the classical limit of some consistent quantum theory. The second difficulty is that, even as a classical theory, general relativity is deficient as a theory of spacetime because it predicts the existence of singularities.

The singularity theorems of Hawking and Penrose [1,2] assert that a spacetime is geodesically incomplete provided that there is a reasonable sense of causality, that the generic condition holds, that there is either a trapped surface or a general cosmological expansion, and that the timelike convergence condition holds. The latter is the requirement that

$$R_{\mu\nu} k^\mu k^\nu \geq 0$$

is satisfied for arbitrary timelike vectors $k^\mu$. In particular, the theorems imply that the spacetimes associated with both gravitational collapse and

† This essay received the joint fifth award from the Gravity Research Foundation, 1993—Ed.
1 Physics Department, Indiana University, Bloomington, Indiana 47405, USA
2 Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK
cosmological expansion are geodesically incomplete. In the standard examples, the Schwarzschild solution or the Friedmann–Robertson–Walker universes, this incompleteness arises as a consequence of the infinite curvature encountered along some spacelike surface. However, the singularity theorems guarantee that this is a generic problem rather than some difficulty arising from over-restrictive assumptions in the derivation of these particular solutions.

The physical meaning of geodesic incompleteness is that a geodesic terminates at a finite proper time in the past, in the future, or both. An observer moving along such a geodesic would reach the boundary of spacetime. The problems posed by this apocalyptic prediction of general relativity are insurmountable, at least within the context of classical physics.

The physical reason for the singularity theorems is that gravitation is universally attractive. Consider a congruence of timelike geodesics parametrized by proper time s along the curves $x^\mu(s)$ with tangent vector $k^\mu = dx^\mu/ds$. The volume expansion $\theta = \nabla^\mu k_\mu$ of the congruence satisfies the Raychaudhuri equation [2]

$$\frac{d\theta}{ds} = -R_{\mu\nu}k^\mu k^\nu + \ldots$$

The right-hand side of this equation consists of effectively negative quantities, with the exception of the first term. The first term is also negative provided that the energy-momentum tensor of matter obeys certain plausible conditions. Using the Einstein equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}$$

we see that eq. (1) holds if

$$(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu})k^\mu k^\nu \geq 0.$$ 

This is the strong energy condition, satisfied for most forms of macroscopic classical matter.

One might hope that an underlying quantum-mechanical description of gravitation would resolve the issue of classical singularities. The only candidate theory available at present appears to be string theory, which is based on the idea that the fundamental structure of an elementary object is a two-dimensional world sheet (rather than the one-dimensional world line of point particles) together with the principle of conformal invariance. If the string world sheet $\Sigma$ has a metric $\gamma_{ab}$, carries coordinates $\xi^a$, $a = 1, 2,$