Sewing Polyakov Amplitudes I: Sewing at a Fixed Conformal Structure

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Abstract. We consider the problem of reconstructing the correlation functions of a conformal field theory on a surface \( \Sigma \) from the correlation functions on a surface \( \Sigma' \) obtained from \( \Sigma \) by cutting along a closed curve. We show that under quite general conditions, the correlation functions on the cut surface can be "sewn" by integrating over appropriate boundary values of the fields.

I. Introduction

In quantum field theory, one ordinarily begins with a Lagrangian and derives a perturbation expansion and Feynman rules. In string theory, this process has been reversed. We have an elegant set of Feynman rules, given by the Polyakov path integral; but despite numerous attempts to write down a field theory of closed strings, a generally accepted formulation does not yet exist. It is thus natural to ask whether information about a field theory can be obtained from the Polyakov path integral.1 In particular, we may ask whether it is possible to derive higher order terms in the perturbation expansion—path integrals over higher genus surfaces—from lower order terms. This is the "sewing" problem.

The sewing problem consists of two distinct elements. The first may be called sewing at a fixed conformal structure. We start with a string world sheet with a given conformal structure, and cut it along a curve to form a new (possibly disconnected) world sheet, which inherits a conformal structure from the original surface. We can then attempt to reconstruct the Polyakov measure on the original world sheet from the measure on the cut surface. If it is possible, such a reconstruction will imply strong relationships between determinants and Greens functions on the two surfaces. More generally, we may start with an arbitrary conformal field theory, and attempt to reconstruct the partition function and

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1 The idea of using the Polyakov path integral to develop a string field theory was first discussed by Tseytlin [1]
correlation functions on the original surface from corresponding quantities on the
cut surface.

The second element of the sewing problem is the sewing of transition amplitudes.
We now start with the full (off-shell) Polyakov amplitude, already integrated over
moduli, for a surface with two or more boundary curves; our goal is to construct
the amplitude for the surface obtained by identifying a pair of boundaries. This
task is more difficult; the identification map between the two boundary curves is
no longer uniquely specified, and the relationship between the moduli spaces of
the cut and sewn surfaces must be understood. Nevertheless, if a closed string field
theory exists, it should be possible to find a procedure for sewing amplitudes.

In this paper, we address the first question of sewing at fixed conformal structure.
We demonstrate that it is possible to sew arbitrary correlation functions for a wide
variety of conformal field theories by functional integration over boundary values
of the fields. Sonoda [2] has given an indirect argument for this result; we take
the more direct approach of explicitly proving the required relationships among
determinants and Greens functions. A subsequent paper will discuss the second
aspect of sewing, the sewing of Polyakov amplitudes. A preliminary announcement
of this work has appeared in [3].

2. Sewing at Fixed Conformal Structure

We start with a Riemann surface $\Sigma$, and cut along a curve $C$ to form a new,
possibly disconnected surface $\Sigma'$ (see Fig. 1). Our goal is to show that correlation
functions for a conformal field theory on $\Sigma$ can be obtained from the corresponding
correlation functions on $\Sigma'$ by functional integration over the boundary values of
the fields on $C$: schematically,

$$\langle \phi \cdots \phi \rangle_{\Sigma'} = \int [d\phi_{|C}] \langle \phi \cdots \phi \rangle_{\Sigma}. $$

In one sense, this relation is obvious. Correlation functions on $\Sigma$ can be obtained