These notes describe some recent developments in the analysis of the possibilities for Pareto improvement when, in a competitive environment, financial markets are incomplete. The basic framework is a general methodology for investigating the welfare effects of policy or institutional changes in equilibrium models, when these can be characterized in terms of perturbing the solutions to a system of smooth equations. Two particular scenarios are discussed in detail: First, the situation where a central government can intervene in the form of wealth taxes and subsidies, and second, the situation where the financial institutions can be modified by increasing the number of available instruments.

1. Overview

The starting point for these notes is the observation that, with incomplete financial markets — or, more generally, with imperfect (in the sense of the Arrow-Debreu-McKenzie benchmark) financial markets — competitive allocation will not, in general, be Pareto optimal. This basic fact suggests a number of questions concerning the possibilities for Pareto improvement, either by direct government intervention (for example, in the form of taxes and subsidies), or by indirect institutional modification (for example, in the form of increased financial opportunities). In the following sections I elaborate on the state of our

* While working in this general area I have benefitted substantially from interaction and collaboration with several of my sometime Penn colleagues or students, Alex Citanna, Atsushi Kajii, Marcos Lisboa and Antonio Villanacci. Conversations at various points with Ronel Elul, John Geanakoplos, Heracles Polemarchakis and, especially, Paolo Siconolfi have also been extremely helpful and illuminating. None of them, however, is responsible for my biases and idiosyncrasies. These notes were prepared for a presentation at the AMASES Conference held in Pugnochiuso, Italy during September 25-28, 1995.
knowledge concerning both possibilities. My plan is pretty standard. In Section II I describe the canonical model of incomplete financial markets, and in Section III two of its most important structural features. Then, in Sections IV and V, I outline a straightforward procedure for analyzing the possibilities for Pareto improvement (in fact, applying to any Walrasian-like model of an economy) and sketch how it has been applied for two leading cases.

A more complete account of this material can be found in the introduction for and contributions to the forthcoming Economic Theory Mini-Symposium on “Pareto Improvement in Incomplete Financial Markets.”

2. Model

I consider a standard model of a competitive, pure-exchange economy with incomplete financial markets. The time horizon is finite, and uncertainty is represented by $S < \infty$ states of the world. Although the formalization encompasses any finite-horizon economy, I will—as is now conventional—focus on the two-period case, where $s = 0$ is today, and $s > 0$ are future states of the world. I assume that all the information in the economy is publicly available. There are $C$ commodities or goods indexed by $c$, with $C > 1$, $I$ instruments or assets indexed by $i$, with $I \geq 0$, and $H$ households indexed by $h$, with $H > 1$. The commodity (and endowment) space is taken to be $\mathbb{R}_+^G$, where $G = C(S + 1)$. A typical household’s preferences are represented by the utility function $u_h : \mathbb{R}_+^G \to \mathbb{R}$, which is assumed to be smooth (i.e., for my purposes, at least $C^3$), differentially strictly increasing (i.e., $Du_h(x_h) > 0$ for $x_h \in \mathbb{R}_+^G$) and differentiably strictly concave (i.e., $\Delta x_h^T Du_h(x_h) \Delta x_h < 0$ for $x_h \in \mathbb{R}_+^G$ and $\Delta x_h \neq 0$), and to have the closure of its indifference surfaces contained in $\mathbb{R}_+^G$. The space of households’ endowments is $E = (\mathbb{R}_+^G)^H$. The space of households’ utility functions is $U = U^H$, where $U$ is the appropriate subset of the $C^3(\mathbb{R}_+^G, \mathbb{R})$ mappings, endowed with the subspace topology induced by the compact-open topology assigned to the whole space. I will describe an economy as an element of the pair $E \times U$, endowed with the product topology. By $x_h \in \mathbb{R}_+^G$, $b_h \in \mathbb{R}^I$, $p \in \mathbb{R}^G$ and $q \in \mathbb{R}^I$ I denote the consumption bundle and the asset portfolio for household $h$, the commodity price vector and the asset price vector, respectively. It will be convenient to take quantity vectors as columns, and price vectors as rows.

The financial structure is represented by an $(S \times I)$-dimensional matrix of asset yields $Y$ expressed in terms of a numeraire commodity, which I take to be the last at each spot $s$, i.e., $c = C$. I will assume that $S > I$, so that markets are incomplete (and, when considering financial innovation, that, $S > I + k$)

1 I do not impose any additional restrictions on the form of the households’ utility functions, for example, that they satisfy the von Neumann-Morgenstern expected utility hypothesis. However, all the results which I will describe can be extended (under slightly stronger assumptions on the economy’s financial structure) to cover such more economically motivated specifications.

2 Alternatively, asset yields $Y$ can be interpreted as being specified in terms of units of account spot-by-spot. The subsequent analysis then applies for any fixed choice of numeraires spot-by-spot, in particular, the choice $c = C$, all $s$. Note that in this case, after appropriate