THE INTERDEPARTURE-TIME DISTRIBUTION FOR EACH CLASS IN THE $\sum_i M_i / G_i / 1$ QUEUE

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Abstract

A stationary queueing system is described in which a single server handles several competing Poisson arrival streams on a first-come first-served basis. Each class has its own generally distributed service time characteristics. The principal result is the Laplace-Stieltjes transform, for each class, of the interdeparture time distribution function. Examples are given and applications are discussed.

Keywords: Interdeparture time, M/G/1 queue, Laplace-Stieltjes transform.

1. Introduction

There are both theoretical and practical reasons for studying the interdeparture time process of a given queueing system. From a theoretical viewpoint, such a study reveals the influence of the arrival and service mechanisms on the departure process. From a practical perspective, an accurate characterization of the interdeparture time process is necessary when studying open networks of queues, since departures from a given node contribute to the arrival process at others.

A general review of work in this area can be found in Daley [4], but several key results will be mentioned here. The first is due to Burke [2], who showed that the interdeparture times from a stationary M/M/c queue are independent of each other, and exponentially distributed (see for example, Gross and Harris [5], page 201.) Later Takács [12], page 80) obtained the Laplace-Stieltjes transform (LST) of the interdeparture time distribution in a stationary M/G/1 queue. Nain [9] extended this result to the case of M/G/1 preemptive-resume priority queues. Marshall [8] obtained an expression for the squared coefficient of variation (the variance divided by the square of the mean, denoted by CV2) of the interdepar-
ture time in a stationary GI/G/1 queue in terms of the (generally unknown) mean delay.

This expression formed the basis for approximations by Kuehn [7] and Whitt [13], [14] for the interdeparture time CV2. The goal in [7] and [13] is to approximate the average delay in open queueing networks. The delay approximation at each queue involves the first two moments of the aggregate arrival and service time distributions. The CV2 of the aggregate arrival process is determined by formulae to handle the superposition, departure, and splitting processes involved. The solution of the system is iterative in [7], whereas it is achieved in [13] by solving a set of linear equations.

A frequent modelling approach in these kinds of approximations for open networks of queues has been to combine all arrival streams into a single stream for the purposes of the departure computations, although there have been recent developments which determine per-class approximations (see Bitran and Tirupati [1], Whitt [15], and Segal and Whitt [10]). In addition, Stanford [11] used a per-class approach to handle priority queueing networks. This paper provides an exact characterization, through the Laplace-Stieltjes transform, of the interdeparture time distribution for each class in the $\sum M_i / G_i / 1$ queue; namely, a single-server queue serving several Poisson arrival streams, with (possibly) distinct service time characteristics. The proposed approach is useful, for instance, when modelling CPU behaviour, where different messages are routed to different queues depending on message type. The exact results derived here are also useful in that the numerical results for the interdeparture time CV2 which we present can be used as a benchmark against which candidate approximations for $\sum G_1 / G_i / 1$ queues can be judged.

The paper does not address, however, the related question of dependence. Since successive interdeparture times are not usually independent random variables, such a study is pertinent in order to more fully understand the interdeparture time process.

2. Mathematical preliminaries

It is assumed that the queue handles traffic from $n$ competing Poisson arrival streams, each with its own service time characteristics, on a FCFS basis. From a given customer's perspective, however, there are only two classes: the one to which the customer belongs, and the "other" class, consisting of all other classes combined. The "other" class is seen by the given customer as a Poisson stream whose arrival rate is the sum of the individual arrival rates of the other streams, and whose service time distribution is the composite of the individual service time distributions weighted by the fraction of customers to come from that class. Consequently in what follows, there are only 2 Poisson streams: the one which the selected customer comes from (class 1), at rate $\lambda_1$, with service time cumula-