Two-Reduction of the Super-KP Hierarchy

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Abstract: Recursion relations are established for the residues of fractional powers of a two-reduced super-KP operator making use of the Baker-Akhiezer function. These show the integrability of the two-reduced even (or bosonic) flows of the super-KP hierarchy. Similar recursion relations are also proven for the residues of operators associated with the odd (or fermionic) flows of the Mulase-Rabin super-KP hierarchy. Due to the presence of a spectral parameter and its fermionic partner in the Baker-Akhiezer function, these recursion relations should be relevant to any attempt to prove or disprove a recent proposal that the integrable hierarchy underlying two-dimensional quantum supergravity is the Mulase-Rabin super-KP hierarchy.

1. Introduction

A surprising link has been established between the quantum theory of two-dimensional gravity and integrable hierarchies of nonlinear equations of the KdV type [1]. It is therefore of considerable interest to discover whether an integrable system underlies two-dimensional quantum supergravity and its topological counterpart. Recent study of a plausible superloop equation [7] has suggested that the Mulase-Rabin super-KP (SKP) hierarchy [11, 6] (or a reduction thereof) is a contender for this role. Further study of this system is therefore warranted.

In this paper, a two-reduced SKP operator is studied. Making use of the Baker-Akhiezer function of the Mulase-Rabin SKP hierarchy, identities and recursion relations satisfied by fractional powers of the reduced operator are established, which show that the two-reduced even (or bosonic) flows of the SKP hierarchy are consistent and Hamiltonian. This Hamiltonian structure has been found previously by Oevel and Popowicz [3] using the Lax formulation of the two-reduced even flows. However, the approach taken in this paper is perhaps more relevant to any attempt to prove or disprove the speculation that the integrable system underlying two-dimensional quantum supergravity is a reduction of the Mulase-Rabin SKP hierarchy [7, 8]. This is because of the appearance of a spectral parameter and its fermionic partner via the Baker function, as discussed further in the conclusion.
In Sect. 2, the Mulase-Rabin SKP hierarchy is briefly reviewed, along with a super-residue formula which is central to the results of this paper. This is applied in Sect. 3 to the proof of recursion relations for residues of fractional powers of a two-reduced SKP operator, and these are used to show the Hamiltonian integrability of the two-reduced even (or bosonic) flows of the SKP hierarchy. Using the Baker function approach, it is also possible to establish identities and recursion relations for the residues of operators which appear in relation to the odd (or fermionic) flows of the Mulase-Rabin SKP hierarchy. This is done in Sect. 4. The conclusion contains an appraisal of the significance of these results to any attempt to establish a link between two-dimensional supergravity and the Mulase-Rabin SKP hierarchy. Calculational details are contained in the appendices.

2. The Mulase-Rabin SKP Hierarchy

The SKP hierarchies of nonlinear equations are formulated in terms of the super pseudo-differential operator

\[ Q = D + \hat{q}_0(X) + q_1(X)D^{-1} + q_2(X)D^{-2} + \ldots \]  

(2.1)

defined on a (1/1) superspace with coordinates \( X = (x, \theta) \). Here, \( D = \partial_x + \theta \partial_{\bar{x}} \) and \( D^{-n} (n > 0) \) is defined by \( D^{-2n} = \partial^{-n} \) and \( D^{-(2n+1)} = D\partial^{-(n+1)} \), where \( \partial^{-n} = \partial_{\bar{x}}^{-n} \) is the usual pseudo-differential operator. Hatted superfields are Grassmann-odd. If the superfields \( \hat{q}_0(X) \) and \( q_1(X) \) satisfy the constraint \( \hat{D}\hat{q}_0 + 2q_1 = 0 \), then [2] there exists a pseudo-differential operator

\[ S = \mathbb{1} + \hat{s}_1(X)D^{-1} + s_2(X)D^{-2} + \ldots \]  

(2.2)

such that

\[ \hat{Q} = SDS^{-1}. \]  

(2.3)

The even (or bosonic) flows of the SKP hierarchy parameterized by the Grassmann-even parameters \( t_n, n \geq 1 \), can be given in the Lax form

\[ \frac{\partial \hat{Q}}{\partial t_n} = [(\hat{Q}^{2n})_+, \hat{Q}] = [\hat{Q}, (\hat{Q}^{2n})_-], \]  

(2.4)

where \((\hat{Q}^{2n})_+\) denotes the differential-operator part of \( \hat{Q}^{2n} \) and \((\hat{Q}^{2n})_- = \hat{Q} - (\hat{Q}^{2n})_+\). In terms of \( S \) the flows are expressed

\[ \frac{\partial S}{\partial t_n} = -(S\partial^n S^{-1})_- S, \]  

(2.5)

and are integrable [2] in the sense that \[ \left[ \frac{\partial}{\partial t_n}, \frac{\partial}{\partial t_m} \right] S = 0. \]

The SKP hierarchy admits several different types of integrable odd (or fermionic) flows (see [6] for a review of these and an explanation of their geometric significance). Here, the Mulase-Rabin hierarchy [11, 6] is considered, although the techniques of this paper can be applied to the Manin-Radul hierarchy [2] with minor modification. The Mulase-Rabin flows with respect to Grassmann-odd parameters \( \tau_{n-\frac{1}{2}}, n \geq 1 \), are given by

\[ \frac{\partial S}{\partial \tau_{n-\frac{1}{2}}} = -(S\partial^{n-1} \partial_{\theta} S^{-1})_- S. \]  

(2.6)