Interpolation and Trend Analysis: Two Geohydrological Applications

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This paper presents two examples of applications to geohydrological data of recent techniques of trend analysis and interpolation. The first case study concerns the resistivity of spring waters of a region in the Appenine mountains. This study has been carried out by Idrotecneco on behalf of the Italian Geological Survey, as part of a pilot study for a geohydrologic map of Italy. The second example concerns the water table levels of an aquifer in the Po River alluvial plain. KEY WORDS: geostatistics, hydrology, contouring.

THE PROBLEM AND THE TECHNIQUE

Given a quantity distributed in space, whose magnitude is known at a few points of the space, one may address himself to one of the following problems:

(1) To predict the values of the quantity at some other point. This can be called interpolation.

(2) To reconstruct the general trend of the quantity, overlooking local variations. In other words, if one considers the quantity as a stochastic function of space, one wants to estimate its expectation function. This is called trend analysis.

The second problem is solved by regression techniques: the expectation function is given some polynomial expansion whose coefficients are estimated either by the least square method or by taking into account the covariance structure of the observations as well.

In the latter case one deals with the following linear model

$$Ey = X'\beta, \quad \Sigma_y = \Sigma$$

where $y$ is the vector of the observations, $\beta$ is the vector of the coefficients of the polynomial, and $X'$ is a full rank matrix whose $i$th row contains the powers of the coordinates of the point at which $y(i)$ has been observed; $\Sigma_y$ means the covariance matrix of $y$.

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Given this model the following estimator of $\beta$:

$$\hat{\beta} = (XB^{-1}X')^{-1}XB^{-1}y$$

can be shown to be unbiased, with minimum variance in the class of the linear estimators of $\beta$ (see Scheffé, 1959).

The covariance matrix of $\beta$ is given by

$$\Sigma_{\hat{\beta}} = (XB^{-1}X')^{-1}$$

The variance of the trend at a point is then given by $a \Sigma_{\hat{\beta}} a'$, where $a$ is a vector containing the powers of the coordinates of the point.

Once the coefficients of the polynomial expansion of the trend have been determined, one may predict the value of the deviation from the trend — i.e., the residual — at any point by multivariate techniques and obtain an interpolation by summing the predicted residual to the estimate of the trend.

The technique used to predict the value of the residuals at a set of points consists in computing the conditional expectation of these residuals given the values of the residuals actually observed at other points.

This conditional expectation is given (Anderson, 1958) by:

$$E(R_1|R_2 = r_2) = \Sigma_{12} \Sigma_{22}^{-1} r_2$$

where $R_1$ is the vector of the residuals, which one wants to predict, and $R_2$ is the vector of the residuals whose realizations $r_2$ one observed. $\Sigma_{22}$ is the covariance matrix of $R_2$, while $\Sigma_{12}$ is the cross-covariance matrix of $R_1$ and $R_2$.

The conditional covariance of $R_1$ given $R_2$ is given by

$$\Sigma(R_1|R_2 = r_2) = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$$

where $\Sigma_{11}$ is the covariance matrix of $R_1$.

An abundant literature is available on this subject. We limit our references here to Matheron (1969), Watson (1971), Agterberg (1974), and Torelli (in press). In particular, Torelli's paper aims to demonstrate that the different treatments of the subject made by various authors can be reduced to the same theoretical basis.

The intent of the present paper is to describe two applications of these techniques to geohydrological problems.

**THE PERGOLA STUDY**

A pilot study for a geohydrological map of Italy has been commissioned by the Italian Geological Survey from Idrotecneco. One 1:50,000 scale geohydrological map was to be produced. The selected map was the No. 291