SOIL MECHANICS

APPROXIMATE ACCOUNTING OF ZONES OF PLASTIC DEFORMATION IN THE BED OF A RIGID PLATE

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The calculation of a strip foundation is proposed when zones of the soil's limiting state are present in the soil bed. A comparison with experiments, which indicated their satisfactory agreement, is made. A comparison with recommendations of the Construction Rules And Regulations is given. A sample calculation is presented.

The results of experimental investigations with rigid footings constructed on sandy and clayey beds indicate that the contact-pressure diagram assumes a clearly expressed saddle-shape pattern with finite stresses along the edges and a parabolic configuration in the limiting state. At the same time, use of a contact model for the design of rigid foundations [1] leads to a uniform distribution of reaction pressure in the median section of the lower surface.

The saddle-shape pattern of the pressure diagram and its transformation are explained by the presence in the bed of simultaneously elastic (sublimiting) and plastic (limiting) regions and the variation of their size under increasing external load. Determination of the stress—strain state of a bed, which possesses the indicated regions with the boundaries between them, the latter being displaced under load, is referred to the category of the mixed problem of the theory of elasticity and limiting equilibrium of soils. Solution of the nonlinear problem (in which conversion from elastic to plastic deformations does not occur immediately on attainment of a certain stress state, as in the mixed problem, but gradually) also leads to a diagram of the same pattern.

V. A. Florin was the first to propose an approximate accounting of the plastic (more precisely, nonlinear, since unloading is not considered) deformations in soil, which results in a saddle-shape configuration of the contact curve. He used S. I. Belzetskii's equation to determine the bearing capacity of the bed, and also the assumption according to which the state of equilibrium of all soil particles may exist only in that case when the configuration of the diagram of pressure against the soil does not project beyond the configuration of the limiting curve corresponding to the limiting state under the entire lower surface of the structure [2]. Regions of limiting equilibrium are considered by truncating the edge pressure ordinates, as determined by Sadovskii's equation in accordance with the theory of a linearly deformable medium.

Different alternate schemes for the approximate accounting of zones of a limiting stress state under a foundation have been proposed in [3-6 and others]; in this case, plate settlements were excluded from consideration.

The simplest and most convenient scheme of accounting for regions of plastic deformations in the bed of a rigid plate has been proposed by Schultz [7], who demonstrated that Sadovskii's stress distribution is valid in the median section of the lower surface, and the pressures do not exceed limiting values near the edges. The area of the contact curve remains constant for a given load; the stresses should therefore increase at the center of the foundation. The problem of defining the pattern of the pressure curve for this scheme (Fig. 1) reduces to the solution of two equations:

\[ \frac{P_0}{\pi \sqrt{a^2-x^2}} = p_0 - x_1 (p_0 - p_{cr})/a, \]

\[ P/2 = \frac{P_0}{\pi} \int_0^{x_1} \frac{dx}{\sqrt{a^2-x^2}} + \int_{x_1}^a \frac{p_0 - x_1 (p_0 - p_{cr})/a}{\sqrt{a^2-x^2}} dx. \]

which are written as applies to a trapezoidal limiting curve for the lower surface of a strip footing. Here, \( P_{cr} \) and \( P_{0\infty} \) are the critical (according to Puzyrevskii—Gersevanov) and maximum pressures along the axis of the foundation’s symmetry, and \( x_1 \) is the boundary separating the sublimiting and limiting curves; it characterizes the extent to which the regions of limiting equilibrium have developed and varies from \( a \) (when \( P = 0 \)) to zero when \( P = P_{II} \). In this case, the bearing capacity \( P_{II} \) of the bed and the pressure \( P_{0\infty} \) are linked by the relationship

\[
P_{0\infty} = P_{II}/a - P_{cr}
\]

Equation (1) is written from the coupling condition on the boundary \( x = x_1 \) of the stresses determined in accordance with Sadovskii’s solution for a linearly deformable medium and the limiting curve

\[
\begin{align*}
\rho_{x\infty} &= \rho_{0\infty} - x\rho_{0\infty} - P_{cr}/a, \quad x \geq 0.
\end{align*}
\]

The second equation (2) corresponds to the plate’s equilibrium. The values of \( P_1 \) and \( x_1 \) are unknown, and can be determined from the solution of Eqs. (1) and (2).

The substitution of (1) in (2) after its preliminary integration yields the relation

\[
P/2 = \frac{\sqrt{a_0 - x^2}}{2} [\rho_{0\infty} - x(\rho_{0\infty} - P_{cr}/a)] - \frac{1}{2a} \rho_{0\infty}(a^2 - x_1^2) - \frac{1}{2a} \rho_{0\infty}(a^2 - x^2)
\]

between \( P \) and \( x_1 \), and the unknown force \( P_1 \) is calculated directly from (1) in accordance with the equation

\[
P_1 = \pi \sqrt{a_0 - x^2} [\rho_{0\infty} - x(\rho_{0\infty} - P_{cr}/a)].
\]

With subsequent integration of the pressure curve, it is possible to write

\[
\begin{align*}
Q_x &= -Q_0 + \frac{P_1}{\pi} \int_0^{x} \frac{dx}{\sqrt{a^2 - x^2}}, \quad (0 \leq x \leq x_1); \\
Q_x &= -Q_0 + \frac{P_1}{\pi} \int_0^{x_1} \frac{dx}{\sqrt{a^2 - x^2}} + \int_{x_1}^{x} \rho_{0\infty} dx, \quad (x_1 \leq x \leq a)
\end{align*}
\]

for the transverse forces and

\[
\begin{align*}
M_x &= M_0 - Q_0 x + \frac{P_1}{\pi} \int_0^{x_1} \frac{x - x_1}{\sqrt{a^2 - x^2}} dx, \quad (0 \leq x \leq x_1); \\
M_x &= M_0 - Q_0 x + \frac{P_1}{\pi} \int_0^{x_1} \frac{x - x_1}{\sqrt{a^2 - x^2}} dx + \int_{x_1}^{x} \rho_{0\infty} (x - x_1) dx, \quad (x_1 \leq x \leq a)
\end{align*}
\]

with the transverse forces and moments.

Fig. 1. Pattern of contact stresses with limiting curve of trapezoidal configuration: 1) for \( P_1 \); 2) for \( P \).