At the same time the method used is regular with respect to the number of nodes of the lattice (with respect to the number of spaces multiplied) and is completely meaningful as $\mathbb{N} \rightarrow \infty$ [16].

LITERATURE CITED


THE $K^0$-FUNCTOR (GROTHENDIECK GROUP) OF THE INFINITE SYMMETRIC GROUP

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The Grothendieck group $K_0(\sigma_\infty)$ of the group of finite permutations of a countable set is described. All semifinite characters of $\sigma_\infty$ are described and with their help the cone of representations $K_+^0(\sigma_\infty)$ is characterized.

0. Introduction. Formulation of the Question; Structures in the $K^0$-Functor

The category of projective modules of the complex group algebra of a finite group exhausts the category of all modules over this algebra, and hence its Grothendieck group ( $K^0$-functor), i.e., the Abelian group of all classes of virtual projective modules with distinguished element ("regular representation"), is isomorphic with the group of virtual characters and thus carries information about all representations of the algebra (group). For infinite and in particular locally finite groups, the projective modules over the group algebra in general constitute only a very small part of all modules, so one could show that the $K^0$-functor in this case does not give sufficient information about the group and its representations. Sometimes this is actually so; for example, the $K^0$-functor of the group $C^*$-algebra of a free group with two generators is only $\mathbb{Z}$ altogether; cf. [15]. It is all the more interesting that for locally-finite groups, and in particular, for the group $\sigma_\infty$ of finite permutations of the natural numbers, the $K^0$-functor contains the richest information.

about the group, its representations, traces, etc., sufficient, for example, to determine the
group algebra uniquely. More precisely, the map of functors

$$C[G] \mapsto K^0(G)$$

associating with the complex group algebra its $K^0$-functor as an Abelian group with dis-
tinguished subsemigroup of faithful projective modules and a distinguished element, the one-
dimensional free module, is an equivalence of the functors $C[\cdot]$ and $K^0(\cdot)$. This
assertion, which is a rephrasing of a more general theorem about AF-algebras [7], makes
the calculation of the $K^0$-functors of locally-finite groups an important problem.

Part of the studies of the theory of the infinite symmetric group $\Sigma_\infty$ (cf. [1, 2])
which the authors have made in recent years is the complete calculation of $K^0(\Sigma_\infty)$, which
is given in the present paper. It turns out that $K^0(\Sigma_\infty)$ and many structures in it play
a central role in the interrelation of questions connected with the group $\Sigma_\infty$. Here the
supply of structures on $K^0(\Sigma_\infty)$ is richer than for the limiting groups $K^0(\Sigma_n)$; for
example, $K^0(\Sigma_n)$ is not a ring like $K^0(\Sigma_\infty)$.

We shall briefly describe the results of the paper. The basic result is the explicit
description of $K^0(\Sigma_\infty)$ and the determination of the cone of faithful modules. First of
all, in $K^0(\Sigma_\infty)$ there is a natural ring structure. This fact singles out an important
subclass of the class of locally finite algebras, which has not been considered previously
and was first mentioned by the authors in [2]. The group algebras of inductive limits of
certain classical groups are involved in this.

Hence for the answer it is convenient to use the spectrum of $K^0(\Sigma_\infty)$ as a ring, and
in particular, the positive spectrum (maximal ideals which are positive on the cone). It
turns out that $K^0(\Sigma_\infty)$ is the ring of functions of formal infinite series in one variable;
the positive spectrum is made up of certain special series which converge in the unit disk and
have the form given in Secs. 1-2. However the cone of faithful modules in $K^0(\Sigma_\infty)$ does
not quite coincide with the cone of nonnegative functions on the positive spectrum, but is
only part of it. Its precise description is the basic result of the paper (cf. Sec. 6). The
general character of the answer is given below in the introduction; it requires using not
only the values of the functions, but also their jets; namely, the values at a point of cer-
tain special differential operators, calculated on the given function.

Now we list the basic structures and properties of $K^0(\Sigma_\infty)$.

1) Limit. $K^0(\Sigma_\infty)$ is the inductive limit of $K^0(\Sigma_n)$ in the category of ordered
Abelian groups with distinguished element.

2) Order. The subsemigroup of faithful modules $K^0_+$ defines on the Abelian group
$K^0(\Sigma_\infty)$ an order with the Riesz property (the so-called interpolation property, slightly
weaker than the lattice axiom):

$$\forall a_1, \ldots, a_n \text{ and } b_1, \ldots, b_m, \ b_j \leq a_i, \ i = i, \ldots, n, \ j = 1, \ldots, m.$$ 

there exists a $c$ with $b_j \leq c < a_i, \ i = 1, \ldots, n, \ j = 1, \ldots, m$.

Cf. below for an explicit description of $K^0_+$. 550