Inflationary Cosmology with a $R + \lambda R_{\mu\nu} R^{\mu\nu}/R$ Lagrangian

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We consider an alternative fourth-order gravity Lagrangian which is non-analytic in the Ricci scalar, and apply it to a Robertson-Walker metric. We find vacuum solutions which undergo power-law inflation. Once matter is introduced the theory behaves very much like ordinary General Relativity, except that the radiation evolution $a \sim \sqrt{t}$ is not allowed since it corresponds to $R = 0$. We comment on the possibility of wormhole solutions in such a theory.

1. INTRODUCTION

Inflationary universe models can be constructed by adding higher order corrections to the usual Einstein-Hilbert Lagrangian for gravity. The first such model was constructed using the trace anomaly of conformally coupled fields which is due to a term $\sim R^2 \log(R/\mu)$ ($\mu$ = renormalization scale) in the Lagrangian [1]. However, typical numbers and types of field in the early universe would not produce enough inflation, and any inflationary phase would finish too abruptly, causing an inhomogeneous universe.

The model was modified by adding a term $\varepsilon R^2$ to the Lagrangian which results in a quasi-de Sitter expansion [2-4]. The Hubble parameter

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1197
decreases slowly for large $\epsilon$ before going into an oscillation phase which can reheat the universe [4,5]. The general Lagrangian so far considered has the form

$$\mathcal{L} = R + \epsilon R^2 + \beta R_{\mu\nu} R^{\mu\nu}.$$  \hspace{1cm} (1)

The coefficients $\epsilon$ and $\beta$ have dimensions of $[\text{length}]^2$ and (in order for stability or to prevent tachyonic solutions) certain signs have to be taken. The $\beta$ term is only relevant for anisotropic models since in the isotropic case it just alters the $\epsilon$ coefficient. New gravitational waves have also been found in such Lagrangians [6].

Various other Lagrangians such as $R + \epsilon R^2 + \beta R^3$ and $\exp(\lambda R)$ have been considered and generally give power law inflation [7–9]: instead of exponential expansion there is typically an expansion law $a \sim t^m$. So long as $m > 1$ and expansion occurs for sufficient time the usual problems, which are solved by inflation, can still be solved.

Some papers have appeared concerning the stability and solutions of general Lagrangians $f(R)$ [11–14]. It is sometimes claimed that such theories or their quasi-de Sitter solutions are unstable [11,12] but at least classically this is not the case. This was analyzed in detail in [13] for the $R + \epsilon R^2$ theory, and was also found to match ordinary gravity as $\epsilon \to 0$. The so-called small perturbations considered in [11] are actually abnormal: they correspond to a large $dH/dt$ ($H =$ Hubble parameter). In analogy with a scalar field inflationary model this $\dot{H}$ term is like a velocity causing you to shoot up the scalar potential, but it is eventually damped [15].

A recent problem has arisen in the case of Bianchi type I models for the Lagrangian (1) for $\beta \neq 0$ (so including the sign that is stable in the absence of anisotropy). The model is found to collapse due to the anisotropy [16]: this was also found by one of us previously using the equations presented in [17] but not taken too seriously at the time since it conflicted with [18]. Note that this is more restrictive than for the case of scalar field inflation, where spatial gradients can cause collapse [19] but not simply anisotropy [20].

Instead in this paper we consider an alternative higher order derivative theory, first proposed in [21]:

$$\mathcal{L} = R + \lambda \frac{R_{\mu\nu} R^{\mu\nu}}{R} + \tau R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}.$$ \hspace{1cm} (2)

Because the parameters are now dimensionless such a Lagrangian could help incorporate higher derivative theories into variable $G$ ones (e.g.

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4 Some works use the equivalent in four dimensions $R + aR^2 + bC^2$ Lagrangian, where $C$ is the Weyl tensor.