In the case of a closed domain, from the equation
\[ k^2 L + k \eta \sum_{n=0}^{N} (i k \eta)^{2n} \sum_{m=0}^{m=0} \partial_{\nu}^{2n+1}(\mathcal{L}) - \omega \eta = 0, \]
where $\mathcal{L}$ is the length of the boundary of the domain $\Omega$, we determine the eigenvalues of the problem $k = k_{pq}$.

Expansions (2.2), (2.3) are valid also in the case of a variable velocity of the wave propagation. Moreover, similar expansions can be applied for the construction of the eigenfunctions of the exterior of $\Omega[1]$. In this case we obtain the following estimate of the imaginary component of the eigenvalues:
\[ \text{Im}(k_{pq}) \sim - (\text{Re} k_{pq})^{1 - \frac{5}{2} \epsilon}. \]

**LITERATURE CITED**


**RADIATION FROM AN OPEN RESONATOR FORMED BY AXISYMMETRIC, WEAKLY CURVED MIRRORS**

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One considers the eigenoscillations of a resonator formed by axisymmetric, weakly curved mirrors. The constructed eigenfunctions satisfy the Dirichlet condition on the mirrors and the Sommerfeld radiation condition at infinity.

1. We consider a three-dimensional resonator, formed by two mirrors $\Gamma_0$ and $\Gamma_1$, where $\Gamma_0$ is the plane $z = 0$, perpendicular to the resonator axis $0z$, while $\Gamma_1$ is a surface, symmetric relative to the axis $0z$, obtained by rotation of the curve $z = 1 + \varepsilon \tilde{f}(\xi)$, $0 < \varepsilon \ll 1$, around $0z$ (Fig. 1).

The infinitely differentiable function $f(\rho)$, defined for $0 < \rho < a$, $f(\alpha) > 0$, has a local maximum for $\rho = 0$, $f(0) = f^{(2n+1)}(0) = 0$, $n = 0, 1, 2, \ldots$, and at some point $a_1$ the function $f(\rho)$ attains a minimum, $0 < a_1 < a$, $f(\alpha_1) < 0$. On the intervals $(0, a_1), (a_1, a)$ the function $f(\rho)$ is monotone (Fig. 2).

In the present paper, for large values of the wave number $k$ and small $\varepsilon(k\sqrt{\varepsilon} \gg 1)$, one investigates the eigenoscillations of the given resonator, concentrated in the domain between the mirrors $\Gamma_0$ and $\Gamma_1$, possessing an axial symmetry, and with the indicated approximation on the formal strictness level one finds imaginary corrections to the eigenfrequencies, corresponding to the energy radiation in the space. The solution of the problem is based on the approach presented in [1].

We seek a nonidentically zero, quasiclassical (for $k \to \infty$) solution of the Helmholtz equation
\[ (\Delta + k^2) \hat{u} = 0, \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \]
in the domain between the mirrors \( \Gamma_0 \) and \( \Gamma_1 \), satisfying the boundary conditions \( \hat{u}|_{\Gamma_0,1} = 0 \) and the radiation condition at infinity: 

\[
\hat{u} \sim A(\theta, \varphi) r^{-1} \exp \left( i k r \right) \quad \text{for } r \to \infty, \quad r = (x^2 + y^2 + z^2)^{1/2},
\]

\( \theta \) and \( \varphi \) are the angular coordinates. In the cylindrical system of coordinates \((\tilde{z}, \tilde{\rho}, \varphi)\), the dependence on the angle \( \varphi \) is assumed to be the following:

\[
\hat{u}(\tilde{z}, \tilde{\rho}, \varphi) = \exp \left( i \ell \varphi \right) u(\tilde{z}, \tilde{\rho}), \quad -\pi < \varphi < \pi,
\]

where \( \ell \) is an integer. In the present paper we shall assume that \( \ell \gg 1 \).

It is known (see [1-3]) that the eigenvalues of the given problem are complex: \( \kappa = \kappa' + i \kappa'' \); the small negative imaginary part \( \kappa'' \) characterizes the damping with time of the corresponding solution of the nonstationary wave equation due to the radiation in the space. For the imaginary part of the eigenvalue, one obtains in the principal approximation the following asymptotic formula:

\[
\kappa'' \sim -\frac{\beta}{2} \left( 1 + \varphi \right),
\]

where \( \beta = 2(\pi q \sqrt{\epsilon})^{-1} \), \( \ell, p, q \) are large integer quantum numbers such that \( 1 < \ell < \ell_0, \ell \sim q \sqrt{\epsilon}, p \gg 1, q \gg 1 \), \( k = k'_\ell p q \sim \mathcal{O}(q) \). The auxiliary parameter \( \alpha \), belonging to the interval

\[
\left( \frac{1}{2} \frac{\beta^2}{(\ell_0 q)^2} + \delta, \quad \frac{1}{2} \frac{\beta^2}{(\ell_0 q)^2} - \delta \right),
\]

where \( \delta \) is some positive constant, is determined from the equation

\[
\left( \frac{1}{2} \frac{\beta^2}{(\ell_0 q)^2} + \delta \right), \quad \left( \frac{1}{2} \frac{\beta^2}{(\ell_0 q)^2} - \delta \right),
\]

\[
q \sqrt{\epsilon} \sum_{j=1}^{\ell_0} \left[ 2 \left( f(\varphi) + \alpha \right) - \frac{\beta^2}{\ell_0^2} \right]^{1/2} d\varphi = p + \frac{1}{2}.
\]

The numbers \( \rho_1(\alpha), \rho_2(\alpha), s = 0, 1, 2 \) are the solutions of the equation

\[
f(\varphi) + \alpha - \frac{1}{2} \frac{\beta^2}{\ell_0^2} = 0,
\]

and the numbers \( \overline{\rho}_1, j = 1, 2, 0 < \overline{\rho}_1 < \overline{\rho}_2 < \rho_1 \) are the solutions of the equation \( f'(\rho) + \beta^2 \rho^{-3} = 0 \); \( \ell_0 \) is such a value of the parameter \( \ell \) for which in the last equation one has \( \rho_1 = \rho_2 \) (Fig. 3).