Ferromagnetism in the Hubbard Model

Examples from Models with Degenerate Single-Electron Ground States

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Abstract. Whether spin-independent Coulomb interaction can be the origin of a realistic ferromagnetism in an itinerant electron system has been an open problem for a long time. Here we study a class of Hubbard models on decorated lattices, which have a special property that the corresponding single-electron Schrödinger equation has \(N_d\)-fold degenerate ground states. The degeneracy \(N_d\) is proportional to the total number of sites \(|\Lambda|\). We prove that the ground states of the models exhibit ferromagnetism when the electron filling factor is not more than and sufficiently close to \(\varrho_0 = N_d/(2|\Lambda|)\), and paramagnetism when the filling factor is sufficiently small. An important feature of the present work is that it provides examples of three dimensional itinerant electron systems which are proved to exhibit ferromagnetism in a finite range of the electron filling factor.

Contents

1. Introduction .............................................. 342
   1.1. Ferromagnetism in the Hubbard Model .................. 342
   1.2. Purpose of the Present Paper ...................... 344
   1.3. Hubbard Models with Degenerate Single-Electron Ground States ........ 345
2. Main Results and Physical Consequences ................... 347
   2.1. Definition ............................................ 347
   2.2. Single-Electron Properties ............................ 348
   2.3. Ferromagnetism for a Special Electron Number ............... 350
   2.4. Ferromagnetism in a Finite Range of Electron Numbers ........... 351
   2.5. Other Properties ....................................... 352
3. Characterization of Ground States ...................... 353
   3.1. Main Theorem ............................................ 353
   3.2. Proof .................................................. 356
4. Percolation Representation ................................. 358
   4.1. Representation ........................................... 358
   4.2. Universal Bounds ...................................... 361
   4.3. Low Density Bounds .................................... 362
   4.4. High Density Bounds .................................... 363
1. Introduction

1.1. Ferromagnetism in the Hubbard Model

In some solids, electronic spins spontaneously align with each other to form strong ferromagnetic ordering. A familiar example is Fe, which maintains long range magnetic order up to the Curie temperature, 1043 K. Given the fact that interactions between electrons in a solid are almost spin-independent, the existence of such a strong order may sound as a mystery. As we shall describe below, this has indeed been an interesting open problem in theoretical physics for quite a long time.

In 1928, Heisenberg [8] pointed out that the spin-independent Coulomb interaction between electrons, when combined with the Pauli exclusion principle, can generate effective interaction between electron spins. Heisenberg’s picture of ferromagnetism was that the relevant electrons are mostly localized at atomic sites, and their spin degrees of freedom interact with each other via “exchange interaction.” It has been realized, however, that his exchange interaction usually has the sign which leads to antiferromagnetic interaction rather than ferromagnetic one. (See [9] for a review.) Nevertheless, Heisenberg’s idea still plays a fundamental role in modern theories of ferromagnetism.

A somewhat different approach to ferromagnetism, which was originated by Bloch [2], is to look for a mechanism of ferromagnetism in which the itinerant nature of electrons, as well as the Coulomb interaction and Pauli principle, play fundamental roles. This project, combined with sophisticated band-theoretic techniques, has led to many approximate theories [10]. A common feature of all these theories is that they are based on the Hartree-Fock approximation (i.e., mean field theory) and its perturbative corrections. Although such approximations can lead to reasonable conclusions in some situations, they have a serious disadvantage from a theoretical point of view. The basic strategy of the approximations is to treat electrons with up and down spins as different species of particles, and then introduce some self-consistency conditions. By doing this, one severely destroys the original $SU(2)$ invariance of the model and gets $Z_2$ invariant self-consistent equations. The existence of ferromagnetism then reduces to a problem of spontaneous breakdown of the discrete $Z_2$ symmetry, which is essentially different from the original problem, a spontaneous breakdown of the continuous $SU(2)$ symmetry. As a consequence, the approximate theories give only two ferromagnetic states with the net magnetization pointing up or down, instead of expected infinitely many states with an arbitrary direction of magnetization. Since a continuous symmetry breaking is a very subtle phenomenon in general, results based on the Hartree-Fock approximation and its improvements are not conclusive enough to answer the fundamental question whether spin-independent Coulomb interaction alone can be the origin of a realistic ferromagnetism in an itinerant electron system. We stress that such a critical point of view has been held by many physicists. See, for example, the review of Herring [10].

Given the subtlety of the problem, it is desirable to have idealized models in which one can develop concrete scenarios for the itinerant electron ferromagnetism. The so-called Hubbard model [11, 13] is a simple but nontrivial model suitable for