Reduced load approximations for loss networks

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In this paper, we present a general scheme with which to view reduced load approximations in loss networks. We use notation motivated by stochastic Petri net (SPN) representations of such models and a technique similar to that described by Ciardo and Trivedi for general SPNs. Previous reduced load approximations have involved link independence assumptions. In our method, we assume independence between sets of links rather than between themselves. Our independence assumptions are thus less drastic than those that have been made previously and better results can be expected. Several examples are given in this context.

1. Introduction

A call arriving at a circuit-switched network requires a circuit from each link along its projected route. If these circuits are available, the call will hold them for its service duration, if not links are used and the call is lost. The circuit-switched network is an example of a loss network in which customers request a set of resources; if these resources are available, the customer holds them for its service time and at the end of this time, releases these resources. If the set of resources is not available, the customer is blocked and lost from the system. A comprehensive account of the applications of loss networks in the analysis of circuit-switched networks can be found in, for example, Girard [13].

Digital packet switched integrated services networks can also be modelled using loss networks. It was this application which motivated the study by Whitt [25]. Packet switched systems use virtual circuit packet switching, blocking a connection if its admission to the system is predicted to sufficiently degrade the quality of service provided to connections already in progress, otherwise the customer is connected and the network’s resources are held by this customer for its service time. Loss networks can also be used to model cellular phone systems (see, for example,
Everitt and Macfadyen [11]), database access problems (Mitra and Weinberger [24]) and, in general, any system in which customers arrive seeking a certain number of resources and who are lost if the resources that they require are not available.

A loss network with fixed routing is known to have a product form equilibrium distribution [1]. However, the complex nature of the state space boundary and the large number of routes over which the product is taken makes the normalising constant impossible to calculate in many cases. Consequently, in most cases, although an invariant measure is known, the equilibrium distribution, and therefore performance measures based on this distribution, cannot be calculated. When this happens, reduced load techniques are commonly used to find approximate performance measures. Such techniques involve making the assumption that loads on separate links are independent, but occur at a rate reduced by the probability of blocking on other parts of a route. When a loss network becomes more realistic, with trunk reservation and alternative routing included, even the product form invariant measure is lost. In these cases, authors such as Girard [13], Heyman [14], Holtzman [16], Katz [18], Kelly [19–21], Hajek and Krishna [15], and Whitt [25] have used reduced load algorithms to derive blocking probabilities and other performance measures for loss networks.

Using SPN models, two papers by Ciardo and Trivedi [6,7] describe a general approach to the analysis of large systems, breaking the original model into independent modules and transporting probabilities and expected values from one module to another to reflect the interdependence between the modules. The result is a system of equations which can be solved iteratively when the relevant equilibrium measures for the modules can be found. Ciardo and Trivedi give a number of examples in their papers, including that of a flexible manufacturing system [7].

In this paper, it is shown how reduced load techniques can be combined with Ciardo and Trivedi’s approach to yield accurate results for large loss networks. To this end, we consider a general class of SPNs which provides generic models for problems in which resources are accessed and used by customers in a variety of ways, and where customers that cannot find the resources they need are lost. In a similar vein to Ciardo and Trivedi [6,7], we postulate an approximation technique which breaks down the original system into a set of simpler modules with interrelated parameters. Whereas Ciardo and Trivedi give examples in which the modules are naturally defined by the problem, our aim is to suggest an approach which takes an arbitrary loss network and leaves analysts free to create their own modular structure based on the parameter values and structure of the particular system. The objective is to use the properties of the system and the insight of the analyst to keep the error in using this approximation procedure to a minimum. When an extreme version of this approach is applied to loss networks, the technique becomes the Erlang fixed point method (EFPM), see, for example, Kelly [19]. In the examples discussed herein, the accuracy of the results can be improved significantly by making less stringent assumptions than those imposed in the EFPM.