A Class of Plane Symmetric Dust Solutions

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A class of plane symmetric solutions containing dust is considered. It is argued that, however inhomogeneous the mass distribution, matter on each plane of symmetry has no net attraction to matter on other planes. It is shown that the geodesic distance between two thin sheets of dust, separated by a particular Kasner region, is zero a finite time after the initial singularity, quickly reaches a maximum, and then decreases as $t^{-1/3}$.

1. INTRODUCTION

The purpose of this paper is to clarify the physical interpretation of the class of dust solutions given using comoving coordinates by the line element

$$ds^2 = dt^2 - t^{-2/3}[b(z) + c(z)t]^2dz^2 - t^{4/3}(dx^2 + dy^2)$$

where $b(z)$ and $c(z)$ are arbitrary functions. Although it is possible to use the freedom in $z$ to put either $b(z) = 1$ or $c(z) = 1$, provided they are non-zero, for convenience we will retain both functions at this stage. The space-time represents a pressure-free perfect fluid with an initial singularity at $t = 0$ and density given by

$$8\pi\rho = \frac{4c}{3t(b + ct)}.$$
This is a well known solution [7], given originally by Ellis [2]. When $c = 0$ it reduces to the vacuum Kasner solution with exponents $\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}$. It is the spatially flat FRW solution (the Einstein–de Sitter model) when $b = 0$. If both $b$ and $c$ are constant (or are proportional) it is a spatially homogeneous Bianchi type I solution.

In the above form, the coordinates are comoving and the fluid flow is orthogonal to the flat hypersurfaces given by $t =$ constant. However, it is generally not spatially homogeneous. It admits three Killing vectors which act on two-dimensional orbits, defining plane symmetry. The solution may be characterized by the property that the surfaces of constant $z$ have vanishing extrinsic curvature.

The general solution was considered by Tomimura [11] who showed that it represents an anisotropic space-time which evolves towards the homogeneous Einstein–de Sitter model for large $t$. Conversely, Zakharov [13] considered plane perturbations of the spatially flat FRW universe. In the linear approximation, he found one mode in which the density perturbations increase as $t^{2/3}$ and another in which they decrease as $t^{-1}$. He showed that when $b(z) \ll c(z)t$, which occurs after a sufficiently large time, the above solution corresponds to the decreasing mode.

In order to consider explicitly inhomogeneous fluid distributions, attention will be concentrated here on various choices for $c(z)$, and it will normally be appropriate to choose $z$ such that $b = 1$ everywhere. Near the initial singularity ($ct \ll b$), the mass distribution is proportional to $c(z)$. By taking specific cases, we can consider arbitrary distributions of mass possibly interspersed with vacuum Kasner regions. Intuitively, it might be anticipated that a single thick wedge of matter will contract under its own gravitational field, and that plane sheets of matter will attract each other, at least after they are causally related. However, this does not appear to be the case.

2. SHEETS OF DUST

The general solution above can be used to give an exact solution representing two plane sheets of matter that are separated by a vacuum Kasner region. For this case, we can scale the $z$ coordinate such that $b(z) = 1$ and choose $c$ to be zero except at two values. Such a situation has been considered by Schmidt [10]. In this case, the space beyond each sheet is also described by the same Kasner solution.

It should be pointed out that these sheets of dust are very different from the thin shells of matter that have been considered by many authors following the methods developed by Israel [6] in terms of a discontinuity in