Cosmological PPN Formalism and Non-Machian Gravitational Theories

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Received January 2, 1995. Rev. version November 30, 1995

By turning to a differential formulation, the post-Newtonian description of metric gravitational theories (PPN formalism) has been extended to include cosmological boundary conditions. The dimensionless expansion parameter is the ratio distance $L$ (measured from the center of a selected space region) to Hubble distance $c/H_0$. The aim was to explore the significance and applicability of a Newtonian cosmology and to clarify to some extent its relation to general-relativistic cosmology. It turns out that up to post-Newtonian order two classes of gravitational theories can be distinguished, here called Machian and non-Machian. In a non-Machian theory like General Relativity the dynamics of cosmic objects within a space region $L \ll c/H_0$ is described by the usual PPN metric set up for the objects, without introducing time-dependent Newtonian potentials at the origin of the PPN coordinate system. Such potentials of obviously cosmological origin seem to be required for the majority of (by our definition) Machian gravitational theories (including, e.g., Brans-Dicke). Conditions for a theory to be Machian or non-Machian are given in terms of algebraic relations for the PPN parameters.

1. INTRODUCTION

There appears to be little agreement what Mach's principle actually is — this was my chief impression from a recent conference on Mach's principle at Tübingen (Germany). Should we relate it to the requirement that the inertial mass of a body is determined by the remaining bodies in the

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Universe? Or should we follow one of Barbour's suggestions [2], that theories must perhaps be formulated in terms of relative quantities to become Machian, since we observe relative quantities (positions, velocities) only? Mach himself seems to be of little help in this respect since for almost every formulation of a Machian principle one may find an appropriate citation in his publications.

Another item from Mach's General Store [1] is the rather vague statement that distant parts of the Universe should have some influence on the form of local laws of physics. In a non-Machian gravitational theory a local experiment should therefore be independent of the cosmological environment. In particular, no fields of cosmological origin should have an influence on the local motion of matter, "local" being taken to denote a sufficiently small space region. The purpose of this article is to show that — within a post-Newtonian approximation — metric theories of gravity may be divided according to the requirement of time-dependent potentials for a consistent local description of cosmological models. This kind of dependence on cosmological boundary conditions is used to define Machian and non-Machian theories.

To deal with a fairly general class of metric gravitational theories, the Parametrized Post-Newtonian (PPN-) formalism by Kenneth Nordtvedt and Clifford M. Will will be used. The notation is taken from Misner et al. [11]; otherwise we follow Will's standard book [14].

The PPN formalism was clearly not designed to deal with cosmology. On the other hand, one has to face the fact that a Newtonian cosmology was developed decades ago (see, e.g., Refs. 7–9,13), with results very similar to those of General Relativity. The relation of this Newtonian cosmology to the Friedman models remained unclear, however. One expects that a Newtonian cosmology should emerge as a first approximation of the general-relativistic theory. Then one may ask for a post-Newtonian cosmology as a second-order approximation. Note the Newtonian cosmology as discussed here is based on a Lorentzian manifold, and hence quite different from the Newton–Cartan formulation of a transition to Newton's gravity (for the latter see, e.g., Refs. 13,6,10,5, and references therein).

Somewhat surprisingly, the PPN formalism as it stands is already able to some extent to deal with cosmological problems, as will be shown subsequently. The keys are (i) to use differential relations to avoid the Minkowskian boundary conditions at spatial infinity in the usual integral formulation of the PPN formalism, and (ii) to take the ratio distance (measured from the center of a selected spatial region) to Hubble distance $c/H_0$ as the expansion parameter. This leads to a simple local description of cosmological models, which should be valid for small values of the expansion