I. The heuristic value of Leśniewski’s mereology

This is the thesis to be upheld. Leśniewski’s conception of collective class and of the relationships between parts and wholes is not an idiosyncratic aberration that leads away from sound classical ideas but is, rather, an open door to a world of fresh, alternative ways of looking at the notion of aggregate. Since axiomatic set theory is by no means a closed book and is still today under intensive examination—particularly stimulated by Cohen’s results on the continuum problem—every approach to the idea of aggregate that helps free the mind from traveling well-worn grooves should be welcome indeed.

From this point of view, it is a disservice to Leśniewski’s originality of outlook to interpret mereology as a Boolean algebra without a zero. Although legitimate from other viewpoints, here such an interpretation would hide more than it reveals. Further, in attempting to pursue the ramifications of Leśniewski’s mereology it does not help to add individual atoms to his theory in order to force it into line with current set-theoretic conceptions. The fact that Leśniewski was not trapped by prevailing atomistic prejudices is very much to his credit; this was definitely an intended position on his part at the time of mereology’s inception, not an omission, even if later on his feelings about the proper role of individuals may have been less assertive—if that is really so.

The relationship of a whole to its parts is a material one that can be abstractly described in many ways not exhaustively covered by the usual set-theoretic relation of inclusion. Frege clearly indicated this in his criticism of Schröder’s confusion of the collective and distributive conceptions of class. Leśniewski put this conceptual distinction to good use and developed a collective theory of classes without fixed elements. And indeed, just as there is no final evidence that the physical universe is made out of some kind of indivisible atoms, neither has it been demonstrated that mathematical classes must be constructed from irreducible elements. In the same way that the “elementary” particles of the physical world are being continually analyzed into ever simpler sub-particles, so can the square be divided without end—to use Frege’s example—into ever smaller squares, or triangles, or any other system of subregions. Although differently motivated, Von Neumann’s continuous geometry
(in which geometric spaces are constructed that have no points at all) falls well within the type of conception alluded to here, a conception which is worth being consciously and systematically explored from all viewpoints. Von Neumann's bottomless space is a good example of the fact that there is nothing absolute about viewing the complex as constructed from the simple, that actually the situation can be developed in reverse. Indeed, it is a more normal process in nature as well as in knowledge to go from the complex to the simpler, then from the simpler to the even more simple. Thus Lesniewski's approach to wholes as logical entities at the same level as parts, his departure from Frege's emphasis on distinguishing an object from the class composed of that object, is a step toward a more concretely descriptive theory of wholes and parts. A forest is not so much a set in Cantor's sense as it is a collective class in Lesniewski's sense; similarly, a melody and a headache are collective classes just as much as their components, a note or a pain.

The basic primitive relationship in mereology is that of part to whole, ingredience. Sets are wholes, totalities of individual parts. However, individuals are not to be thought of as atoms necessarily, but rather as being themselves new totalities of individuals. Every individual is identical with the totality of itself, as well as identical with any totality of individual parts into which the given individual can be analyzed. Individuals and totalities, then, are logical entities at the same level; there is no unbreachable gap between them in the way membership creates an unbreachable entitative distinction between a nonempty set and the singleton composed of that set. Since the role of inclusion in Cantor's set theory is that of relating sets to subsets or supersets, it seems natural to interpret ingredience as ordinary inclusion. This has at least two significant undesirable consequences. One is that it settles the question "What is a part?" by identifying parts with subsets, thus precluding any alternative answers. Such alternative answers are, for example, "A part is any partially distinguishable ingredient of a whole (whether or not the part is fully separable from the whole)," or "A part is any entity internally related to a whole," or "A part is any individual or multiplicity of individuals into which a whole can be analyzed by any definable kind of division," etc. The second undesirable consequence is that the continuum problem becomes inextricably linked with Cantor's set theory, obscuring the significant possibility that the continuum may require an axiomatic treatment independent of set theory. Let us explain further why this second consequence is undesirable (Sections II and III describe alternatives to the first consequence).

If one accepts that the power set of a given set should have a definite cardinality (Cantor's continuum hypothesis and its generalization), then the question "What is a subset?" becomes academic. Since the generalized continuum hypothesis is not set-theoretically provable, the question of what a subset is suddenly acquires a central logical importance. From