1. The original system of Ontology, constructed by Leśniewski in 1920, is based on the functor of singular inclusion as the only primitive ontological term. As regards its meaning, the functor of singular inclusion approximates the meaning of the copula 'is'. In natural languages without indefinite articles, in Latin or in Polish for instance, the approximation appears to be closer than is the case in the languages in which the indefinite articles have a role to play. It is, therefore, not surprising that English or German speaking logicians find Leśniewski's logical language offending their linguistic intuitions, and treat his Ontology, and the theories which presuppose it, with a certain amount of suspicion. They might have been less mistrustful of Ontology, had Leśniewski based it on a different primitive term. It is quite likely that the functor of weak inclusion would prove to be more acceptable at least to those logicians who had been acquainted with the researches of Boole and Schröder.

In the original system of Ontology — I shall refer to it as System $\mathfrak{I}$ — the meaning of the copula 'is' ('$\varepsilon$' in symbols) is determined axiomatically with the aid of the following thesis:

\[ T_1 \ [ab]:: a \varepsilon b. \equiv . \ [\exists c]. c \varepsilon a. [cd]: c \varepsilon a. d \varepsilon a. \Rightarrow . c \varepsilon d .\ [c]: c \varepsilon a. \Rightarrow . c \varepsilon b \]

to be read:

for all $a$ and $b$, $a$ is $b$ if and only if ((for some $c$, $c$ is $a$)
and (for all $c$ and $d$, if $c$ is $a$ and $d$ is $a$ then $c$ is $d$)
and (for all $c$, if $c$ is $a$ then $c$ is $b$))

In this 'translation' of $T_1$ the copula 'is' is underlined to remind us that it is, as it were, a sort of idealisation of the copula as used in ordinary English.

Among the various theses of Ontology we have:

\[ T_2 \ [a]: \exists x(a). \equiv . [\exists b]. b \varepsilon a \]
i.e., for all $a$, there-exists-at-least-one $a$ if and only if for some $b$, $b$ is $a$

\[ T_3 \ [a]: \exists \cdot x(a). \equiv : [bc]: b \varepsilon a. c \varepsilon a. \Rightarrow . b \varepsilon c \]
i.e., for all $a$, there-exists-at-most-one $a$ if and only if for all $b$ and $c$, if $b$ is $a$ and $c$ is $a$ then $b$ is $c$
T4 \([ab] . a \subseteq b \iff [c]: cz.a \supseteq .czb\)

i.e., for all \(a\) and \(b\), all \(a\) is \(b\) if only if for all \(c\),

\[
\text{if } c \text{ is } a \text{ then } c \text{ is } b
\]

Now, it is easy to see that in the light of \(T2-T4\) the axiom of system \(\mathfrak{A}\) can be said to be inferentially equivalent to

T5 \([ab]: a \subseteq b \iff .ex(a). sol(a), a \subseteq b\)

i.e., for all \(a\) and \(b\), \(a\) is \(b\) if and only if (there-exists-at-least-one \(a\), there-exists-at-most-one \(a\), and all \(a\) is \(b\))

The significance of \(T5\) lies in that it reveals, in a way, the meaning of \(T1\).

The logical researches in which Leśniewski, Tarski and Sobociński were engaged in the twenties resulted in successive simplifications of the axiomatic foundations of System \(\mathfrak{A}\), and eventually enabled Leśniewski to establish, in 1929, that a system of Ontology could be based on a single axiom as simple as the following thesis:

T6 \([ab]: a \subseteq b \iff [\exists c]. a \subseteq c. c \subseteq b\)

i.e., for all \(a\) and \(b\), \(a\) is \(b\) if and only if for some \(c\),

\[
(a \text{ is } c \text{ and } c \text{ is } b)
\]


In what follows I propose to give a similar account of the developments in the study of the axiomatic foundations of systems of Ontology with the functor of weak inclusion as the only primitive ontological term.

2. In the original system of Ontology the functor of weak inclusion is usually defined with the aid of \(T4\). It was in the early fifties that, prompted by a discussion with Professor Woodger, I became interested in this functor and particularly in the possibility of using it as the only primitive term of a system of Ontology. Still within the framework of System \(\mathfrak{A}\) I was able to deduce

T7 \([a]: ex(a) . = .[\exists b]. \sim (a \subseteq b)\)

T8 \([a]: sol(a) . = . . [bed] . : b \subseteq a . c \subseteq a . \supset: b \subseteq c . \lor . c \subseteq d\)

and

T9 \([ab]: a \subseteq b . . . [\exists c] . \sim (a \subseteq c): a \subseteq b . . [cde] . : c \subseteq a . d \subseteq a . \supset: c \subseteq d . \lor . d \subseteq c\).