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LITERATURE CITED

TECHNICAL FLUCTUATIONS IN A QUARTZ CRYSTAL OSCILLATOR WITH AUTOMATIC GAIN CONTROL

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Technical fluctuations in quartz crystal oscillators with automatic gain control (AGC) are studied. Using the example of a three-terminal capacitive transistor circuit, the contribution of technical fluctuations of individual components to the spectral characteristics of the output signal is determined. It is shown that an AGC system can significantly improve the oscillator technical characteristics.

1. Fluctuation Analysis of the Oscillator

We will consider an oscillator with AGC using the three-terminal capacitive circuit of Fig. 1 of [6]. We omit thermal and shot noise sources and use Kirchhoff's equations to describe the circuit. For the independent variable we use the transistor base-emitter input voltage (x), and take the transistor characteristic in the form of Eq. (4) of [6].

We consider nonlinearity of the quartz crystal [7] and relative fluctuations in the elements Lc, Rc, C, Ce, s and s, which we denote by ρL(t), ρR(t), ρc(t), ρce(t), ρs(t) and ρs(t), while (ρL) ≪ 1 and (ρR) ≪ ω^2(ρL) for

any $\lambda$, where $\lambda$ are the various subscripts. After expansion of Kirchhoff’s equations in $x$, we obtain the following equation of motion

$$
\dddot{x} + \omega_0^2 \dot{x} = F(x, \dot{x}, \ldots) + G(\mu_{AGC}, x, \dot{x}, \ldots) + \Phi(t, x, \ldots).
$$

(1)

All notation was described in [6] and $\Phi(t, x, \ldots) = \sum_k \Phi_k(t, x, \ldots)$;

$$
\Phi_k(t, x, \ldots) = -\dddot{x}_k - \frac{s_k}{C} \dot{x}_k - \frac{s_k}{C} \dddot{x}_k,
$$

$$
\Phi_R(t, x, \ldots) = -\frac{u_0}{Q_c} \dddot{x}_R - \frac{u_0 Z_0^2}{Q_c C} \dddot{x}_R - \frac{u_0}{Q_c} \dddot{x}_R - \frac{u_0 Z_0^2}{Q_c C} \dddot{x}_R,
$$

$$
\Phi_x(t, x, \ldots) = -\dddot{x}_R - 3 \dddot{x}_R - 3 \dddot{x}_R - \dddot{x}_R - \dddot{x}_R - \dddot{x}_R - \dddot{x}_R - \dddot{x}_R,
$$

$$
\Phi_{ce}(t, x, \ldots) = -\frac{z_0 C_c}{C_c} \dddot{x}_{ce} - \frac{z_0 s_0}{C_c} \dddot{x}_{ce} - \frac{z_0}{C_c} \dddot{x}_{ce} - \frac{z_0 s_0}{C_c} \dddot{x}_{ce},
$$

(1a)

$$
\Phi_{24}(t, x, \ldots) = -\frac{s_{24}}{C_c} \dddot{x}_{24} - \frac{s_{24}}{C_c} \dddot{x}_{24} - \frac{s_{24}}{C_c} \dddot{x}_{24} - \frac{s_{24}}{C_c} \dddot{x}_{24},
$$

$$
\Phi_{11}(t, x, \ldots) = -\frac{s_{11}}{C_c} \dddot{x}_{11}, \quad Q_c = \frac{u_0 L_c}{R_c},
$$

$$
C_c = \left(\frac{1}{C_c} + \frac{1}{C_c} \right)^{-1},
$$

and $C_\Sigma$ is the equivalent capacitance of the crystal circuit.

A solution of Eq. (1) at $\Phi(t, x, \dot{x}, \ldots) = 0$ is known [6, 8], and for $\Phi(t, x, \dot{x}, \ldots)$ not equal to zero we will seek a solution in the form of Eq. (9) from [6].

Before proceeding to the solution of Eq. (1), we will use the results of [6, 8] to generalize the equation obtained to the case of arbitrary self-oscillating systems with an AGC circuit which has more than one degree of freedom. In doing this we will assume, as usual [8], that the technical fluctuations of the parameters are sufficiently small in comparison to their stationary values, and that the fluctuations are sufficiently slow in comparison to the fundamental oscillation frequency. In this case, as can easily be shown, the equation of motion for an oscillator with ideal AGC system [6] can be represented in the form

$$
\hat{L} \dddot{x} = F(x, \dot{x}, \ldots) + G(\mu_{AGC}, x, \dot{x}, \ldots) + \Phi(t, x, \dot{x}, \ldots),
$$

(2)

where $\Phi(t, x, \dot{x}, \ldots)$ is a random effect produced by technical fluctuations of oscillator elements.

Using the methods developed in [6, 8], from Eq. (2) we can find the fluctuation equations:

$$
\dot{a} = p_z a + \frac{a_z q_z(t) + a_z q_z(t)}{\delta},
$$

$$
\dot{q} = -q_z a + \frac{a_z q_z(t) - a_z q_z(t)}{\delta},
$$

(3)

where $q_z(t)$ and $q_z(t)$ are defined in [8].

The system thus obtained, Eq. (3), determines the spectra of amplitude-frequency fluctuations, which differ from the analogous values of [8, 9] in that instead of the usual values of nonisochronicity ($q$) and limiting cycle strength ($p$) there appear everywhere the nonisochronicity ($q_\Sigma$) and limiting cycle strength ($p_\Sigma$) of the generator with AGC [6]. Comparing the spectra obtained with the spectra of technical fluctuations in amplitude and frequency of a conventional oscillator with many degrees of freedom, we find that the presence of an AGC system may lead to a significant reduction in the spectrum of amplitude and frequency fluctuations and a change in their form, because of $p_\Sigma$ and $q_\Sigma$. 

1235