Seminormal stratified default theories

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In this paper we study seminormal default theories. The notions of stratification and strong stratification are introduced. The properties of stratified and strongly stratified default theories are investigated. We show how to determine if a given seminormal default theory is strongly stratified and how to find the finest partition into strata. We present algorithms for computing extensions for stratified seminormal default theories and analyze their complexity.

1. Introduction

Reiter's default logic is among the best-known and best-motivated of the formalisms for describing commonsense reasoning [10]. Given a default theory, its defaults serve as constraints that restrict the class of theories a reasoning agent might adopt as possible sets of consequences of the initial theory. Since this formalism is meant to handle incomplete information we allow a default theory to have multiple sets of consequences, usually referred to as extensions, or no such sets at all.

However, problems such as finding all extensions of a given default theory or determining whether a given default theory has at least one extension are NP-hard or NP-complete even for very simple classes of default theories [7, 8].

In this paper we study a class of seminormal default theories for which extensions can be computed more easily than in the general case. There are several ways to do it based on syntactic restrictions on formulas and default theories [5, 7]. For example, Kautz and Sellman [7] investigated the class of seminormal disjunction-free default theories, that is default theories in which only seminormal defaults were allowed and all the formulas appearing in defaults were required to be conjunctions of literals.

In this paper we investigate seminormal default theories. We do not require that all formulas in defaults are disjunctive-free. We allow arbitrary seminormal defaults but we impose more restrictive conditions on dependencies between defaults.

We will adopt the concept of stratification, which was used in the context of logic programming and various nonmonotonic modal logics [1, 2, 4, 6, 11] to the case of seminormal default logic. Informally, a default theory is stratified if its set of defaults can be partitioned into "strata". Defaults which have a common propositional variable in their consequents must appear in the same strata. Moreover, all defaults having in their consequent a variable which appears in the proper justification of a
default $d$ must appear in an earlier strata than $d$ and those defaults having a common variable with the prerequisite of $d$ may not appear in later strata.

We prove that every strongly stratified seminormal default theory has an extension, and that each ordering of defaults which agrees with a strong stratification generates an extension. Moreover, each extension for such theory is generated by some ordering which agrees with a stratification and such extensions can be found by expanding the theory stratum by stratum. We examine properties of stratified seminormal theories and show that many well known properties of normal default theories hold also for them.

Finally we describe algorithms for computing extensions for seminormal stratified default theories. It is shown that if a given seminormal default theory can be partitioned into several strata the number of calls to propositional provability procedure needed to compute the extensions can be significantly reduced. We show that checking if a given seminormal default theory is stratified or strongly stratified and computing the finest partition into strata is easy and can be done in polynomial time. The same approach of cycle detection in a suitably defined graph which was presented in [8] in the case of autoepistemic theories works also here.

2. Preliminaries

In this section we introduce the basic notions and recall relevant results. We study default theories over propositional language $\mathcal{L}$. For any propositional formula $\varphi$ by $\text{Var}(\varphi)$ we denote the set of all propositional variables which appear in $\varphi$.

**Definition 1.** A default $d$ is called **seminormal** if it is of the form

$$d = \frac{\alpha : \beta \land \gamma}{\gamma},$$

where $\alpha, \beta, \gamma$ are arbitrary propositional formulas, called prerequisite, proper justification and consequent of $d$ (respectively). Formula $\beta \land \gamma$ is called the justification of $d$.

A default theory $(D, W)$ is **seminormal** if and only if every default of $D$ is seminormal.

**Definition 2.** Let $D$ be a set of seminormal defaults. A function $\text{rank}$ assigning an ordinal number to every default from $D$ is a stratification function for $D$ if for any $d, d' \in D$, where

$$d = \frac{\alpha : \beta \land \gamma}{\gamma} \text{ and } d' = \frac{\alpha' : \beta' \land \gamma'}{\gamma'},$$

the following three conditions hold:

1. if $\text{Var}(\gamma) \cap \text{Var}(\gamma') \neq \emptyset$ then $\text{rank}(d) = \text{rank}(d')$, and