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REMARKS ABOUT SYLLOGISTIC WITH NEGATIVE TERMS

The present paper deals with the axiomatic systems of the traditional logic (syllogistic) of I. Thomas, A. Wedberg and C. A. Meredith (see [7]). Besides, a new axiomatic system of the traditional calculus of names is presented here. This system differs — as I know — from all hitherto constructed axiomatic systems of syllogistic.

The systems of Thomas, Wedberg and Meredith are based on the two-valued propositional calculus. The Aristotelian “a” (all... are...) and the sign of nominal negation (i.e. negation of nominal arguments) “’” are primitive terms of the first and second system. The sign of nominal negation and the functor “e”, forming universal negative propositions, are primitive terms of Meredith’s system. Each of these three systems has different set of axioms and primitive rules of inference but they are equivalent (see [7], p. 310). Hence I confine here to Wedberg’s system in which the rule of detachment and the rule of substitution for nominal variables are primitive rules of inference.

Wedberg’s system is based on the following set of axioms and definitions:

AW1. \( X a X' \)
AW2. \( X' a X \)
AW3. \( X a Y \rightarrow Y' a X' \)
AW4. \( X a Y \land Y a Z \rightarrow X a Z \)
AW5. \( X a Y \rightarrow \sim X a Y' \)
DW1. \( X e Y \equiv X a Y' \)
DW2. \( X i Y \equiv \sim X e Y \)
DW3. \( X o Y \equiv \sim X a Y \).

Wedberg’s system includes Łukasiewicz’s axiomatic system of syllogistic ([5], p. 87) and the known classical theses with negative terms. Hence we obtain in this system not only the laws of the square of opposition, the laws of conversion and the categorical syllogisms but also the laws of obversion, contraposition and inversion. It seems however that Wedberg’s axioms do not characterize sufficiently the traditional theory of categorical propositions. In particular, they do not exclude the interpretation of nominal negation which may be presented graphically as follows:

\[
\begin{array}{c}
X \\
\hline
X' \\
\end{array}
\]
Hence, it is not excluded here that some names are not contradictory but only contrary with respect to their negations. It can be shown however that — under such an interpretation of nominal negation — the meaning of some categorical propositions differs considerably from the meaning of these propositions in the current language.

Let us assume that $D = \{N^+, N^-, P^+, P^-\}$, where

- $N^+$ is the set of odd positive integers,
- $N^-$ is the set of odd negative integers,
- $P^+$ is the set of even positive integers,
- $P^-$ is the set of even negative integers.

Let us denote by "$\subseteq$" the relation of the inclusion of sets and by "$F$" the function of the complement of a set defined as follows:

$$(a) \quad F(N^+) = N^-, \quad F(N^-) = N^+, \quad F(P^+) = P^-, \quad F(P^-) = P^-.$$ 

It can be easily seen that all axioms of Wedberg's syllogistic are satisfied in the modal $\mathfrak{M} = \langle D, \subseteq, F \rangle$ is "$a$" and """ are interpreted as the names of the relation $\subseteq$ and of the function $F$, respectively.

The sets $N^+, N^-, P^+, P^-$ are mutually exclusive. On the basis of the definitions DW1, DW2 and the equalities (a) the following propositions are true in the model $\mathfrak{M}$: “Some odd positive integers are even negative integers”; “Some even positive integers are odd negative integers”. Hence some particular affirmative propositions containing as subject and predicate two mutually exclusive non-empty names are true in this model.¹

As it has been remarked Wedberg’s system contains all laws of the traditional calculus of names. However, Wedberg’s set of axioms does not characterize sufficiently the constants of the Aristotelian syllogistic and the sign of nominal negation. In particular — as it has been shown by the example above presented — Wedberg’s system does not exclude the interpretation of some categorical propositions which is not in accordance with the sense of current language or with some known interpretation of these propositions. It seems that Wedberg’s system and the equivalent systems of Thomas and Meredith should be strengthened especially by the axioms which exclude the above presented interpretation.

In this paper I attempt to formulate such axioms. The system presented here differs from Wedberg’s system among others by the fact that it is based on the first order functional calculus without identity. The number of the axiomatic systems of syllogistic (with nominal negation and without such a negation) is considerable and therefore the construction of new axiomatic system of this kind should be justified.

The categorical propositions are commonly used in the current language and — as it seems — in accordance with the traditional theory of categorical propositions. It is also known that the traditional logic — in spite of its similarity to fragments of some contemporary logical calculi — can not be treated as a part of Łeśniewski’s

¹ It can be easily shown that the system of Meredith also does not exclude such an interpretation of particular affirmative propositions. It is sufficient, in order to show it, to replace in the model $\mathfrak{M}$ the relation $\subseteq$ by the relation $\subseteq^*$ defined as follows:

$$X \subseteq^* Y \equiv X \subseteq F(Y).$$