LIMIT ANALYSIS OF FRAME SYSTEMS STIFFENED BY PANELS*
Gaetano Zingone** and Pasquale Mancuso**

SUMMARY: Limit analysis of frame systems stiffened by perforated and perforated panels by discretisation of the panels into one-dimensional finite elements. The fundamental equation of the problem arrived at is very general in scope and is of particular interest for the analysis of the structural-systems subjected to shear as the effect of the wind or of the earthquake one.

1. Introduction and approach to the problem.

In a previous paper presented to the AIAS Congress in June 1973 [7] we analysed the elastic behaviour flat of frame systems stiffened by perforated panels subject to shear. As explained there, the topic is of particular interest when erecting such structures in seismic areas. In order to assess the strength of such systems we felt we should pursue the investigation in the limit condition. The problem was approached by discretising the stiffener panel into one-dimensional finite elements. When the panel has been divided into m fields and n planes, we arrive at a squared meshed network structure, having m • n redundant bars, connected to the frame. This mode of division allows us to take the location and size of the hollow into account without rendering the calculation unduly laborious. It is thus possible to follow the rigidity of the system as the dimensions of the opening vary, and comply with the boundary conditions. The collapse load is obtained as the lower limit of the class of all the values for the possible mechanisms.

2. Limit analysis of the stiffening panel.

With reference to the panel element shown in Fig. 1, let us analyse the limit state under the action of a horizontal shear force applied to the free border of the element.

If \( a_f \) denotes the reinforcement per unit length of the transverse section of the element under study, the translation equilibrium along \( y \) is

\[
x = \frac{1}{\sigma_0 \cdot t + \sigma_{yo} \cdot a_f + \frac{\sigma_0 \cdot t \cdot \varepsilon}{2}} (\sigma_{yo} \cdot a_f + \frac{\sigma_0 \cdot t \cdot \varepsilon}{2})
\]

(1)

the contribution of the compressed reinforcement being negligible and the plasticisation of the zone under tension being extended as far as the neutral axis.

If we assume

\[
\mu = \frac{\sigma_{yo} \cdot a_f}{\sigma_0 \cdot t}
\]

(2)

(1) may be written more simply

\[
x = \frac{1}{1 + \mu} \left( \frac{\varepsilon}{2} + \mu \cdot t \right)
\]

(3)

From the rotation equilibrium we obtain

\[
M = \frac{\sigma_0 \cdot t}{2} \left( \frac{\mu \cdot t}{1 + \mu} - \frac{\varepsilon}{4} \right) \left( -4 \left( \frac{1}{1 + \mu} \right) \right)
\]

(4)

Using Stassi D’Alia’s yield criterion, which for a biaxial stress state assumes the following expression:

\[
\sigma^2 + 3\tau^2 + \left(1 - \frac{1}{\varrho} \right) \sigma \cdot \sigma_0 = \frac{\sigma_0}{\varrho}
\]

depending on the characteristic parameter \( \varrho = \sigma_0/\sigma_0' \),

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** Istituto di Scienza delle Costruzioni, Facoltà di Ingegneria, Università di Palermo.
defined by the ratio between the compressive and tensile limit stresses of concrete [6], we easily obtain

\[ T = \frac{\pi \cdot \sigma_0 \cdot \xi \cdot s}{8 \sqrt{3}} \left( 1 + \frac{1}{\eta} \right)^2 \]  

(5)

and hence

\[ \xi = \frac{8 \sqrt{3} \cdot T}{\pi \cdot \sigma_0 \cdot s \left( 1 + \frac{1}{\eta} \right)^2} \]  

(6)

which, substituted in (4) supplies the relation between the two characteristics \( M \) and \( T \)

\[ \frac{M}{M_0} + \frac{4}{3\pi^2} \left( 4 + \frac{1}{\mu} \right) \frac{T^2}{T_0^2} = 1 \]  

(7)

in which we put

\[ M_0 = \frac{\sigma_0 \cdot s \cdot l}{2} \left( 1 + \frac{1}{\eta} \right)^2 = m_0 \cdot l^2 \]

\[ T_0 = \frac{\sigma_0 \cdot s \cdot l}{2 \sqrt{3}} \left( 1 + \frac{1}{\eta} \right) = t_0 \cdot l . \]  

(8)

Observing that the moment in the constrained section is

\[ M = T \cdot b \]

from compliance with the yield condition (7) we obtain the collapse load \( T_r \)

\[ T_r = t_0 \frac{\sqrt{b^2 l_0^2 + \frac{16}{3\pi^2} \left( 4 + \frac{1}{\mu} \right) m_0 l^2 - b t_0}}{\frac{8}{3\pi^2} \left( 4 + \frac{1}{\mu} \right) m_0} . \]

(9)

3. Discretisation of the panel into one-dimensional elements.

On the assumption of a discretisation of the panel into one-dimensional finite elements already analysed Fig. 2, for the mechanism shown in Fig. 3 it turns out that

\[ S_0 = S_0' = \frac{d}{2l} T_r \]  

(10)

where

\[ d = \sqrt{l^2 + b^2} \]

substituting (9) in (10)

\[ S_0 = S_0' = \frac{t_0 d}{4} \left[ \frac{b^2 l_0^2 + \frac{16}{3\pi^2} \left( 4 + \frac{1}{\mu} \right) m_0 l^2 - b t_0}{\frac{8}{3\pi^2} \left( 4 + \frac{1}{\mu} \right) m_0} \right] . \]

(11)

4. Limit analysis of the whole frame-panel system.

Let us consider a system consisting of a frame with a constant limit moment stiffened by a panel. The panel is divided into \( m \) fields and \( n \) planes; each of the resulting elements, of width \( l = L/m \) height \( b = H/n \) and thickness \( s \), is discretised into one-dimensional elements as stated earlier (Fig. 4).

The global collapse scheme of the system assumed is as in Fig. 5 and the corresponding collapse load is:

\[ F_0 = \frac{1}{H} \left( 4M_0 + 2mnS_0 \frac{l \cdot b}{d} \right) \]  

(12)

where \( M_0 \) denotes the limit moment of the frame elements being \( H = nb \), from (12) we have

\[ F_0 = 4 \frac{M_0}{nb} + 2 \frac{mn}{nb} S_0 \frac{lb}{d} , \]