A New Class of Gas-Kinetic Relaxation Schemes for the Compressible Euler Equations

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Starting from the gas-kinetic model, a new class of relaxation schemes for the Euler equations is presented. In contrast to the Riemann solver, these schemes provide a multidimensional dynamical gas evolution model, which combines both Lax–Wendroff and kinetic flux vector splitting schemes, and their coupling is based on the fact that a nonequilibrium state will evolve into an equilibrium state along with the increase of entropy. The numerical fluxes are constructed without getting into the details of the particle collisions. The results for many well-defined test cases are presented to indicate the robustness and accuracy of the current scheme.

KEY WORDS: Gas-kinetic relaxation schemes; compressible Euler equations.

1. INTRODUCTION

Many high-resolution shock-capturing schemes have been developed in the past 20 years. Most of them either attempt to resolve wave interactions through the upwind biasing of the discretization or explicitly introduce numerical viscosity in just the amount needed to resolve discontinuities.(6) Generally, there are different design principles for the construction of high-resolution schemes, such as Jameson's symmetric limited positive (SLIP) formulations.(7) The following analysis is similar to van Leer's terminology,(17) but with a different viewpoint, which contains more physical intuition rather than mathematical manipulation.

A high-resolution scheme usually consists of two parts, the reconstruction of the initial data and the dynamical evolution started from the
constructed data. In other words, these two stages can be regarded as geometrical and dynamical correlations for the gas flow around an artificially defined cell boundary.

The geometrical stage is a kinematic description of the flow variables, which tries to recover the continuous physical reality from the discretized data. Caution should be taken whenever doing the interpolations, otherwise Gibbs phenomena or local extrema can be unphysically created. The inclusion of some kind of limiters to eliminate the creation of any local extrema or to restrict the large subcell variations is physically correct and numerically necessary. The interpolated quantities could be continuous or discontinuous at the cell boundary according to the real flow situations; both cases are equally important. The behavior of different limiters is usually problem dependent: one limiter which is good for one test case may show unfavorable results for another one. It seems rather pointless to indulge in this variation of the limiters without good motives. However, using a high-order interpolation does not guarantee that the final scheme will also have the same order of accuracy as the order of the interpolation, since obtaining the solutions of the model equations (such as the Euler equations) with the interpolated data as the initial condition is still questionable.

The second stage is the dynamical correlation, which uses physical models to construct the gas evolution picture starting from the interpolated data, and finally to obtain the numerical fluxes at a cell boundary. For the Euler equations, due to the complicated nonlinear wave interaction, the exact solution can be found only for simple initial value problems. For example, the Riemann solver is an exact solution under the condition of two constant states in the left and right sides of a cell boundary for 1D gas flow. Godunov-type schemes are based on this solution. One advantage of the Godunov method is that it includes dynamical interactions in the gas evolution process, and flow correlations can be clearly observed from the approximate Riemann solver, such as Roe's scheme, where Roe's average is actually some kind of correlation of the flow variables from both sides of a cell boundary. However, except under special physical situations, the real flow distributions are not necessarily two constant states. Thus, the Riemann solver is actually a first-order representation of gas dynamical evolution, and the reconstructed data from the first stage have to be abandoned in some way in order to fit the initial value condition of the Riemann solver. Most authors, however, ignore this point, by mixing the order of the interpolation with the order of dynamics. For example, van Leer points out that all one needs to do to raise the order of accuracy of the upwind differencing scheme is to raise the order of accuracy of the initial value interpolation that yields the zone-boundary data. Although