One-Particle Subspaces in the Stochastic XY Model

Yu. G. Kondratiev and R. A. Minlos

Received May 6, 1996; final December 19, 1996

We study the spectrum of the generator $H_\beta$ of the Glauber dynamics for a model of planar rotators on a lattice in the case of a high temperature $1/\beta$. We construct two so-called one-particle subspaces $\mathcal{H}_\pm$ for $H_\beta$ and describe the spectrum of the generator in these subspaces. As a consequence we find time asymptotics of the correlations for the Glauber dynamics.

KEY WORDS: Plane rotators; stochastic dynamics; invariant subspaces; Dirichlet forms and operators.

1. INTRODUCTION AND DESCRIPTION OF THE MODEL

We study here stochastic (Glauber) dynamics for an infinite system of plane rotators in the high-temperature regime. This dynamics (given by a Markov semigroup) is constructed in such a way that a Gibbsian distribution $\mu$ for the system of plane rotators (the so-called XY model) is invariant with respect to the dynamics. By using this semigroup we can define a reversible stationary Markov process with the stationary distribution $\mu$. Such an approach to the study of Gibbsian measures began with works of Dobrushin, Holley, and Holley and Stroock and was developed extensively and intensively in the study of many models of classical and quantum statistical physics, quantum field theory, etc. For a review we refer the reader to refs. 3 and 12.

Let us denote by $H$ the generator of the stochastic dynamics, i.e., the generator corresponding Markov semigroup acting in the space $L^2(\Omega, \mu)$ ($\Omega$ is the phase space of our system, see below). We are interested in the spectral properties of the self-adjoint operator $H$, namely in the structure
of the lower branches of its spectrum. We find two invariant subspaces \( (\mathcal{H}_+ , \mathcal{H}_- ) \) for \( H \) (so-called one-particle subspaces) and describe in detail the spectrum of \( H \) in these subspaces. We also show that the remaining part of the spectrum of \( H \) lies above that spectrum.

From the general point of view we deal here with the wider idea which consists in establishing a quasiparticle picture for the operator \( H \) which controls the dynamics of a system with an infinite number of components with a local, translation-invariant interaction.

The first step is to find the so-called one-particle subspaces \( \mathcal{H}_1 , . . . , \mathcal{H}_k \) for the operator \( H \). These subspaces are invariant with respect to the operator \( H \) and are cyclic with respect to a group of translation operators \( U_s \). We assume that this group is isomorphic to the lattice \( \mathbb{Z}^d \). There exist unitary mappings

\[
V_j : \quad H_j \rightarrow L^2(T^d, d\lambda)
\]

where \( T^d \) denotes the \( d \)-dimensional torus, i.e., the group of characters of the group \( \mathbb{Z}^d \). These mappings transform the operators \( U_s | \mathcal{H}_i \) and \( H | \mathcal{H}_i \) into the multiplication operators by the functions

\[
\exp i(\lambda, s), \quad \lambda \in T^d, \quad s \in \mathbb{Z}^d
\]

and

\[
m_i(\lambda), \quad \lambda \in T^d
\]

respectively. The spaces \( \mathcal{H}_1 , . . . , \mathcal{H}_k \) describe states of quasiparticles (elementary excitations). The function \( m_i(\lambda) \) is the dispersion of a particle of the \( i \)-th kind, i.e., \( m_i(\lambda) \) is the energy of this particle as a function of its quasimomentum \( \lambda \in T^d \).

The present work is devoted to this first step in the picture of the quasiparticle representation. The next step in this picture is a description of the whole system as a "free gas of quasiparticles" as well as an investigation of their "bounded states." This step is described in detail in refs. 14 and 16.

The problem of constructing the one-particle subspaces is well developed in the case of lattice models of quantum Euclidean fields with weak interaction in discrete space-time.\(^{(14)}\)

In the case of continuous space with continuous time the one-particle spaces for the \( P(\phi)_2 \) model were constructed in the pioneering work of Glimm, Jaffe, and Spencer with the help of the cluster expansion for the corresponding Markov field.\(^{(17)}\) In the present paper (as well as in ref. 15) the spectral analysis of Markov field generator with continuous time is