Asymptotic Behavior of the Density for Two-Particle Annihilating Exclusion

V. Belitsky

Received February 21, 1994

We consider a stochastic process which presents an evolution of particles of two types, $A$ and $B$, on $\mathbb{Z}^d$ with annihilations between particles of opposite types. Initially, at each site of $\mathbb{Z}^d$, independently of the other sites, we put a particle with probability $2p < 1$ and assign to it one of two types with equal chances. Each particle evolves on $\mathbb{Z}^d$ in the following manner: independently from the others, it waits an exponential time with mean 1, chooses one of its neighboring sites on the lattice $\mathbb{Z}^d$ with equal probabilities, and jumps to the site chosen. If the site to which a particle attempts to move is occupied by another particle of the same type, the jump is suppressed; if it is occupied by a particle of the opposite type, then both are annihilated and disappear from the system.

The considered process may serve as a model for the chemical reaction $A + B \rightarrow$ inert. Let $\rho(t)$ denote the density of particles in this process at time $t$. We prove that there exist absolute finite constants $c(d)$ and $C(d)$ such that for all sufficiently large $t$, $c(d)t^{-d/4} \leq \rho(t) \leq C(d)t^{-d/4}$ in the dimensions $d \leq 4$ and $c(d)t^{-1} \leq \rho(t) \leq C(d)t^{-1}$ in all higher dimensions. This completes and makes more precise the results obtained by us earlier and shows that asymptotically the density behaves like that in a similar process called two-particle annihilating random walks which was studied by Bramson and Lebowitz. Our proofs are based on the approach developed in their and our works. We use the basic properties of random walk and various tools which have been designed to study simple symmetric exclusion processes.

KEY WORDS: Diffusion-dominated reaction; two-particle annihilating exclusion; asymptotic upper and lower bounds of the density.

1. INTRODUCTION

The process called two-particle annihilating exclusion (abbreviated to AE) was introduced and studied in ref. 2. It was shown there that $\rho(t)$, the
density of particles in the AE at time $t$, is asymptotically bounded from above by $Ct^{-1}$ in the dimensions $d > 4$ and does not exceed $t^{-d/4}t^c$ when $t \geq t(\varepsilon)$ for all $\varepsilon > 0$ in the dimensions $d \leq 4$. The present paper provides an asymptotic lower bound for the density of particles in the AE (Propositions 1 and 3 from Sections 3 and 5, respectively) and improves the upper bound calculated in ref. 2 for $d \leq 4$ (Proposition 2 from Section 4). When combined, these propositions state the following:

**Theorem 1.** Let $\rho(t)$ denote the density of particles in the two-particle annihilating exclusion at time $t$. Then there exist absolute positive finite constants $c(d)$, $C(d)$ such that

$$c(d)t^{-d/4} \leq \rho(t) \leq C(d)t^{-d/4}$$

when $d \leq 4$

$$c(d)t^{-1} \leq \rho(t) \leq C(d)t^{-1}$$

when $d \geq 4$

for all sufficiently large $t$.

The proof of Theorem 1 uses essentially the methodology developed by Bramson and Lebowitz$^{(3)}$ for studying the asymptotic behavior of density in a process similar to the AE which is called two-particle annihilating random walks (ARW). In the ARW, particles of two types, say $A$ and $B$, evolve on $\mathbb{Z}^d$. Each particle executes a (simple symmetric) random walk independently of all other particles and is annihilated and removed from the process when it meets an opposite-type particle; the latter is annihilated as well. The AE is a modification of the ARW obtained by imposing one additional condition: when a particle attempts to move to a site which is occupied by another particle of the same type, this move is suppressed. Thus, in the AE, two particles of the same type never occupy the same site simultaneously. Consequently, we expect the AE to be a more appropriate model of the chemical reaction $A + B \rightarrow$ inert than the ARW (for the relation of the considered processes to chemistry, see Section 1 of ref. 3 and the references therein). Our interest in the AE was motivated by this fact. Also, the upper bound for $\rho(t)$ provided in Theorem 1 is used in ref. 4 for studying the occurrence of a rare event in the exclusion process.

The particles of the same type interact in the AE by the rules of the (simple symmetric) exclusion process (see Chapter VIII of ref. 6 for the definition). In our proofs, we substitute the exclusion process by another process called a stirring system. It is constructed in the following way: to each bond of the lattice $\mathbb{Z}^d$ we attach an alarm clock such that the times when its alarm goes off form a Poisson point process on $[0, \infty)$ with the intensity $(2d)^{-1}$; each time when the alarm goes off at a bond, the contents of the sites connected by this bond interchange (see ref. 5 for the complete