Annals of Operations Research 12(1988)51 –64

SOLVING JIGSAW PUZZLES BY COMPUTER*

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Abstract

An algorithm to assemble large jigsaw puzzles using curve matching and combinatorial optimization techniques is presented. The pieces are photographed one by one and then the assembly algorithm, which uses only the puzzle piece shape information, is applied. The algorithm was experimented successfully in the assembly of 104-piece puzzles with many almost similar pieces. It was also extended to solve an intermixed puzzle assembly problem and has successfully solved a 208-piece puzzle consisting of two intermixed 104-piece puzzles. Previous results solved puzzles with about 10 pieces, which were substantially different in shape.

Keywords

Computer vision, curve matching, jigsaw puzzle assembly, traveling salesman, assignment, pattern recognition, 2-D shape.

1. Introduction

1.1. In this paper, we describe a technique for solving jigsaw puzzles by computer vision and report the experimental results of applying our technique to 104-piece jigsaw puzzles. In our approach, each piece of the puzzle is photographed and digitized and its boundary is calculated. Using only this boundary data, our algorithm calculates a global matching between these puzzle pieces and consequently produces an assembly of the puzzle (as shown in figs. 1 and 5). Similar techniques are applied to a successful solution of an intermixed double puzzle. Two 104-piece puzzles were processed together with no a priori idication that this is the case. The computer program is able to determine that it is dealing with two separate puzzles and then assembles each of them successfully.

*Work on this paper has been supported by Office of Naval Research Grant N00014-82-K-0381, National Science Foundation Grant No. NSF-DCR-83-20085, and by grants from the Digital Equipment Corporation, and the IBM Corporation.

1.2. The jigsaw puzzle assembly problem is regarded as a strenuous test for 2-D curve matching techniques and was studied in [F-G, 6], where a 9-piece puzzle was assembled, using matching techniques based on the Freeman chain code and some heuristics, and in [R-B, 12], where a 4-piece puzzle was assembled using curve matching based on the so-called "boundary-centered polar encoding" technique.

In both these cases, as in our approach, the puzzle is turned upside down, so the only information used is the shape of the pieces (in [F-G, 6] it is called an "apictorial" puzzle). In [R-B, 12], the puzzle pieces were traced on a data tablet. Both [F-G, 6] and [R-B, 12] require a high degree of discrimination between the boundary curves of different puzzle pieces to enable successful puzzle assembly.

1.3. Our approach uses as its basis the Schwartz–Sharir curve matching algorithm (see [S-S, 13]). We use this method to assemble "apictorial" jigsaw puzzles of more than 100 pieces, many of which are similarly shaped (see figs. 3 and 4). We are using commercial children's puzzles (e.g. fig. 1 shows the other side of the "Mickey Mouse & Donald Duck" puzzle by Jaymar), the pieces of which were turned upside down and photographed separately by a black and white camera. This photographic procedure introduces additional noise which does not exist when the input data is obtained artificially.

1.4. Because of these three factors – a relatively large number of puzzle pieces, many strong similarities between boundaries of different pieces, and the additional level of noise – it is difficult to obtain a complete puzzle assembly just from the basic matching algorithm and straightforward heuristics. Hence, the results of the "local" Schwartz–Sharir matching algorithm are taken as input to a second "global" matching which uses these results and assembles the puzzle using further combinatorial optimization techniques. This "global" approach differs substantially from the "local" techniques, which were used in previous works, and enables us to solve puzzles which are by an order of magnitude larger, and which piece shape is much more similar, than in the previous works.

1.5. Since the basic local matching algorithm is described in detail in [S-S, 13], we give only a short overview of it in sect. 4 and this paper is devoted mainly to the description of the global algorithm, which is given in sect. 5, and the double puzzle solution, which is described in sect. 6. However, it should be noticed that the good performance of this local algorithm is an essential component in our puzzle assembly. In fact, from our experiments it is clear that the puzzles with the simpler sizes and shapes used in previous works ([F-G, 6], [R-B, 12]) could be easily assembled using only the local matching algorithm, without any global considerations. Indeed, in a previous experiment, we have managed to assemble a simpler 15-piece puzzle using only local matching.