Planning under Uncertainty using Parallel Computing

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Industry and government routinely solve deterministic mathematical programs for planning and scheduling purposes, some involving thousands of variables with a linear or non-linear objective and inequality constraints. The solutions obtained are often ignored because they do not properly hedge against future contingencies. It is relatively easy to reformulate models to include uncertainty. The bottleneck has been (and is) our capability to solve them. The time is now ripe for finding a way to do so. To this end, we describe in this paper how large-scale system methods for solving multi-staged systems, such as Bender's Decomposition, high-speed sampling or Monte Carlo simulation, and parallel processors can be combined to solve some important planning problems involving uncertainty. For example, parallel processors may make it possible to come to better grips with the fundamental problems of planning, scheduling, design, and control of complex systems such as the economy, an industrial enterprise, an energy system, a water-resource system, military models for planning-and-control, decisions about investment, innovation, employment, and health-delivery systems.

*Keywords:* Linear programming, planning under uncertainty, large-scale systems, parallel processors, stochastic programming, decomposition principle.

0. IMPORTANCE OF COMING TO GRIPS WITH COMPLEXITY

One approach to understanding complexity is to model large systems such as those of the economy or large-scale enterprises and engineering systems in the form of mathematical systems of linear and non-linear inequality constraints, and then seek a solution using computers that has desirable properties such as

1. Revision of an invited address before the XIIth Mathematical Programming Symposium, August 1985, entitled "Need to do Planning Under Uncertainty and the Possibility of Using Parallel Processors for this Purpose".

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maximizing some objective function. The solutions can then be used as guidelines for decision makers to follow. Parallel processors may make it possible to come to better grips with such problems.

Up to now, systems easiest to model and solve have been those whose resource availabilities, possible choices of technology, and future demands are assumed known with certainty. Industry and government routinely solve deterministic mathematical programs for planning and scheduling purposes, some involving thousands of variables and inequality constraints. As noted in the abstract, the solutions obtained are often ignored because they don't properly hedge against future contingencies. It is relatively easy to reformulate models to include uncertainty. The bottleneck has been (and is) our capability to solve them. The time is now ripe for finding a way to do so. To this end, we describe how large-scale system methods for solving multi-staged systems, such as Benders Decomposition, high-speed sampling, Monte Carlo simulation, and parallel processors can be combined to solve some important planning problems involving uncertainty.

Section 1 describes the classical approach to solving large-scale dynamic systems using nested decomposition and how it can be applied to solving stochastic dynamic problems. Section 2 discusses the computer architecture requirements and configurations in a broad sense that are suitable for decomposition problems. Sections 3 and 4 detail the mathematical structure of the deterministic and stochastic case including, in particular, capacity planning problems under uncertainty. Section 5 discusses a possible application to an Intelligent control system with learning in a multi-stage production system. Section 6 and 7 describe on-going research and final comments.

1. PROGRESS TO DATE SOLVING LARGE-SCALE SYSTEMS
As early as 1970, it became evident that although much progress had been made in proposing methods for solving very large-scale systems, especially those involving uncertainty, these were for the most part ideas on paper - little or no testing had been done on practical problems. Because there was no systematic testing, papers in the literature were in reality little more than academic exercises in pure mathematics. In the 1970's, many places like Stanford's newly formed Systems Optimization Laboratory began to place greater emphasis on systematically testing algorithms on real applications. Research has now reached the stage in which basic software tools are being used for extensive testing and development on deterministic dynamic linear programs. Techniques under study include variants of the simplex method, interior solution methods, and nested decomposition.

While there has been much progress on the deterministic class outlined on the left branch in the above diagram, the many problems involving uncertainty shown on the right branch are by far the most important and require for their

2. DANTZIG, DEMPSTER and KALLIO, eds. [14] can be used as a general reference source of research work on uncertainty; this source also contains 25 pages of references.