Approximating nonstationary $Ph(t)/M(t)/s/c$ queueing systems

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Abstract

Nonstationary phase processes are defined and a surrogate distribution approximation (SDA) method for analyzing transient and nonstationary queueing systems with nonstationary phase arrival processes is presented. Regardless of system capacity $c$, the SDA method requires the numerical solution of only $6K$ differential equations, where $K$ is the number of phases in the arrival process, compared to the $K(c+1)$ Kolmogorov forward equations required for the classical method of solution. Time-dependent approximations of mean and variance of the number of entities in the system and the number of busy servers are obtained. Empirical test results over a wide range of systems indicate the SDA is quite accurate.

Keywords and phrases

Phase process, approximation, queueing, nonstationary.

1. Introduction

This paper considers multiserver finite capacity queueing systems in which the arrival process is a nonstationary generalization of a phase renewal process and the service process is a nonstationary state-dependent Poisson process. An approximation is developed that makes use of Polya-Eggenberger (PE) distributions and surrogate distribution approximation (SDA) methods. SDA methods are reviewed in Taaffe [9] and Taaffe and Clark [10]. The PE distribution is described in Johnson and Kotz [3].

*This research was partially funded by National Science Foundation grant ECS-8404409.

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Neuts [6] describes phase distributions and stationary queueing systems with phase renewal arrival processes. Continuous phase random variables are the time until absorption in a finite state continuous-time Markov process with exactly one absorbing state. The inter-arrival time random variables described in this paper are the time until absorption in a finite state continuous-time nonstationary Markov process with exactly one absorbing state. Such a process is referred to here as a nonstationary phase process and is represented by the symbol $Ph(t)$.

Nonstationary phase processes, defined in the next section, maintain the flexibility of phase distributions and allow for time-dependence. Queueing systems with nonstationary phase arrival processes can be analyzed via numerical integration of large sets of nonhomogeneous first-order linear differential equations. The number of differential equations can, however, be too large for convenient computation if the capacity of the system and the number of phases in the phase arrival process are large.

SDA methods reduce the number of differential equations required to analyze nonstationary queueing systems. Rider [7] approximated the $M(t)/M(t)/1$ system, Clark [2] and Rothkopf and Oren [8] approximated the $M(t)/M(t)/s$ system, all using SDA methods or variations of SDA methods. The SDA method and state space partitioning applied to multivariate queueing systems worked well in the 2-priority nonpreemptive nonstationary single-server system (Taaffe and Clark [10]). In all cases, the number of differential equations required by the approximation is independent of system capacity.

The rest of this paper is organized as follows. First, the nonstationary phase process generalization of phase random variables is defined. The Kolmogorov-forward equations and the moment differential equations (mde's) for the $Ph(t)/M(t)/s/c$ are developed in sect. 3. The SDA method, which is based on the mde's, is then reviewed and extended for $Ph(t)/M(t)/s/c$ systems in sect. 4. Finally, empirical evidence is reported on the accuracy of the SDA method for approximating the time-dependent behavior of the mean and standard deviation of the number of customers in the system.

2. Nonstationary phase processes

This section defines phase distributions using the notation of Neuts [6], then phase distributions are presented in a form that is well suited for use in an SDA approximation. A generalization of phase random variables to nonstationary phase processes is then described.

Consider a Markov process on the finite set of integers $1, 2, \ldots, k+1$ having infinitesimal generator $G = (g_{ij})$, with state $k + 1$ absorbing and all other states transient. Let the vector of initial state probabilities be denoted by

$$v_0 = (\alpha, 0),$$