DECOMPOSITION APPROXIMATION OF QUEUEING-NETWORK CONTROL MODELS WITH TREE STRUCTURES

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Abstract

We devise a decomposition approximation method for a general branching queueing network with service-rate controls. The decomposition method, which reduces and simplifies computation routines considerably, results from the conditions for monotone optimal control policies in induction arguments. We first isolate each branch of connected queues as a subnetwork and then link the subnetworks through branching nodes to approximate the optimal control policies for the global network. The numerical results for a wide class of cost models show that the approximated optimal policies from the decomposed problems are sufficiently close to the optimal policies of the global problem.

Keywords and phrases

Control of queueing networks, decomposition, Markov decision processes.

1. Introduction

In this paper we introduce a decomposition approximation method for solving optimal service rates for general queueing networks with tree structures. A given network will be divided into subnetworks of branches that are numerically more tractable and the subnetworks will then be linked together through branching nodes to generate the approximate optimal control policies for the original global network. The monotone properties of the optimal control policies that usually reduce computation efforts will also be presented. This paper is an initial work in decomposing queueing network control models. Decomposition of the lexicographically ordered state space of queueing networks without necessarily regarding physical structures can be seen in Courtois [1] and the decomposition of large-scale systems other than queueing networks was studied by White and Schlussel [16]. Controlling of service in a queueing network was initially done by Torbett [5]. Subsequent works on various service-
rate control models of queueing networks can be seen in Rosberg et al. [9], Hajek [3], Stidham and Weber [13], and Jo [7]. In this paper we consider a new tree-like queueing-network model and study the approximate behavior of the optimal control policies. In addition, the results of this paper can be of useful contribution to the analyses of the complex network systems such as computer systems, communication networks, and production job shops.

The network control model under consideration is formulated through a semi-Markov decision process. The state of the system is described by the customer distribution throughout the network and each component of the state is the number of customers in each node. The decision for a given state is then to select a service-rate vector, each component of which is the service rate to be prescribed for each node. The objective for our model is to maximize the expected discounted total return of operating the system for a finite planning horizon starting with each given state. The objective function is the sum of holding costs, service costs, and service rewards incurring to all the nodes in the network. We shall use the sufficient conditions for the monotonicity in induction arguments to describe the behavior of the optimal policies as the state varies due to service completion or adding a marginal customer. The resultant monotone properties will next lead to insensitivity of an optimal service rate for a node to state change in some other branch. All the properties of the optimal policies for the finite-horizon problem can be extended to the infinite-horizon problem by standard dynamic programming theories.

The decomposition procedures follow from the properties of the optimal control policies, especially the insensitivity between different branches. The global network will be decomposed into various subnetworks which were formerly branches of the original network. Then we will consider each subsystem separately and compute the optimal service rates for each given state. The optimal service rate of the last node of each subsystem will later be replaced in coordination with its immediate successor nodes after solving a new subsystem containing the branching node and its immediate successor nodes. Then one can compare the global optimal service rates for each state by combining all the subsystems by our method. Numerical results from various cost functions show that such approximated optimal policies are sufficiently close to the true ones and that the computation time is significantly reduced. A case study of implementing our decomposition method for a 5-node queueing network is also shown in this paper.

2. Formulation of network control model

The model under consideration is an open Jacksonian [5] network consisting of \( M \) nodes without cycling and merging. Hence, the network has a tree-like structure. An exogenous arrival will join node \( i, 1 \leq i \leq M \), from an independent Poisson process with rate \( \lambda_{si} \). Upon completion of service from each node, a customer visits