COMPARISON OF HYPOTHESIS TESTING TECHNIQUES FOR MARKOV PROCESSES ESTIMATED FROM MICRO VERSUS MACRO DATA

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Abstract

We estimate the parameters of a Markov chain model using two types of simulated data: micro, or actual interstate transition counts, and macro aggregate frequency. We compare, by means of Monte Carlo experiments, the validity and power for micro likelihood ratio tests with their macro counterparts, previously developed by the authors to complement standard least-squares point estimates. We consider five specific null hypotheses, including parameter stationarity, entity homogeneity, a zero-order process, a specified probability value, and equal diagonal probabilities. The results from these micro-macro comparisons should help to indicate whether micro panel data collection is justified over the use of simpler state frequency counts.

Keywords and phrases

Markov processes, micro data, macro data, estimation, hypothesis testing.

1. Introduction

Despite a rather impressive list of applications of Markov process models to physical and social phenomena (see, for example, Adelman [1], Collins and Preston [10], Telser [32,33], Collins [9], Lever [23], Spilerman [30], Yakowitz [37,38], Trinkl [35], Meredith [27], Shah [29], Kelton and Kelton [16], Kalbfleisch et al. [13], and Kelton [15]), many researchers have been reluctant to employ a stochastic-process methodology. The usefulness of this type of modeling has been generally restricted by the difficulty in performing statistical inference, such as hypothesis tests, from the results.
For empirical implementation of Markov models, there are essentially two types of potentially available data. If one has access to micro data or actual interstate transition counts, maximum likelihood estimators of the stationary or nonstationary state transition probabilities can be computed in a straightforward manner (see sect. 2). These estimators have been shown to be consistent, asymptotically unbiased, and asymptotically normal. Furthermore, likelihood ratio and chi-square hypothesis tests have been developed (Anderson and Goodman [2], Billingsley [5,6], Kullback et al. [20], Basawa and Rao [3]).

On the other hand, micro data are not always available, due to individual privacy or business disclosure rules for social processes or simply due to high collection costs. Instead, we may have to rely on macro, aggregate frequency (for example, census) data, where we know only the number of entities occupying a given state at a given time. Actual state-to-state transitions are not observed. Van der Plas [36] showed that, given irreducibility and ergodicity of the independent Markov chains, the stochastic process of the macro data is also a Markov chain. For the case of macro data, perhaps the most common method of point estimation is restricted least squares (implemented by a standard quadratic programming procedure and reviewed briefly in sect. 3). See Miller [28], Madansky [26], Lee et al. [22], MacRae [25], Kelton [14], Kalbfleisch et al. [13], Van der Plas [36], and Kalbfleisch and Lawless [12] for development and refinement of the least-squares procedure. Bedall [4] proposed certain specialized chi-square tests based on analogy to frequency table analysis techniques. Kelton and Kelton [17–19] proposed a general methodology, following Chow [8], Fisher [11], and Theil [34], for devising hypothesis tests when only macro data are available. The methodology is described in sect. 3. This general framework was used to develop three tests for the adequacy of the stationary Markov chain model (first-order process, stationarity, and homogeneity) as well as to develop tests of a more specialized nature — constant diagonal elements and a probability equal to a specified value. The distributions of the test statistics were investigated in factorially designed Monte Carlo experiments. In general, it was found that treating the test statistics as having F distributions with appropriate degrees of freedom under the null hypotheses of interest led to rejection proportions close to the desired levels. Additional Monte Carlo results indicated favorable power of the proposed tests.

Nevertheless, despite their seemingly good performance, the macro estimation and testing procedures are based on inferior, aggregated data, and it is of some interest to learn whether they compare favorably with estimates and tests based on individual transition counts. Lawless and McLeish [21] examined the loss of information due to aggregation of micro panel data and found that, although the aggregated data were generally less informative than micro data, they still could, in some cases, give rather good estimates and predictions. For the sampling experiments by Lee et al. [22], one notes that the root mean square errors for the restricted least-squares estimates are consistently larger for various sample sizes than for the micro maximum likelihood