ON THE ANALYTIC SOLUTION
OF THE LANE-EMDEN EQUATION

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Received December 3, 1994

The Lane-Emden equation describing temperature in a star in hydrostatic equilibrium was recently solved analytically by N.T. Shawagfeh [1] with highly accurate results using decomposition [1-9]. Further consideration suggests some computational advantage, avoidance of the initial transformations, possible generalizations in astrophysics, and a measurement to determine masses of the cepheids.

Key words: Stellar structure, cepheids, decomposition method, generalization in astrophysics.

1. INTRODUCTION TO THE LANE-EMDEN EQUATION

In the theory of stellar structure, a basic equation on which numerous studies have been made is the Lane-Emden equation [1]. It describes the temperature variation of a star in hydrostatic equilibrium. The equation has the form
\[ r^{-2}(d/dr)r^2(d/dr)T + \lambda T^m = 0. \]

The given conditions are \( T(0) = 1 \) and \( dT/dr\bigg|_{r=0} = 0 \). The \( \lambda \) is a constant proportional to the gravitational constant \( g \) and dependent on the stellar gas described. Values of \( m \) as any real number between zero and five are of interest. Exact solutions for the range of \( m \) have not been found and an important objective is a non-perturbative analytical method which is accurate and easily computable. Such a method is the decomposition method [2-9] which now provides a physically correct solution to a wide variety of physical problems since it does not require smallness assumptions. Linearization and perturbation are the usual procedures which, of course, change the problem supposedly being solved to a convenient one for a mathematical rather than a physical solution. Analytical methods which do not require a change of the model into mathematically more tractable, but necessarily less realistic representation, are of primary concern when correspondence of results with nature is the objective. An "exact" solution obtained by changing the physics is not superior to an "approximate" method which solves the actual nonlinear problem as accurately as necessary. It is an analytical solution rather than a discretized numerical scheme requiring intensive computation. The decomposition method further, is a general method not limited to a special case.

The decomposition method provides realistic solutions by solving the nonlinear problem without simplification. It provides a solution in the form of a very rapidly convergent series such that generally less than one half dozen terms provide a sufficiently accurate solution and remaining error is sharply reduced with a few more terms. As an example, calculation of a general Duffing equation [1] reduced error to less than 0.0001% in four terms. A brief description of the basic method follows for the convenience of physicists who may be unfamiliar with the decomposition procedure.

Beginning with a deterministic equation \( Fu(t) = g(t) \) where \( F \) represents a general nonlinear ordinary differential operator involving both