Optimum Design of Vibrating Cantilevers:  
A Classical Problem Revisited

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Abstract. Optimum design of vibrating cantilevers is a classical problem widely used in the literature and textbooks in structural optimization. The problem, originally formulated and solved by Karihaloo and Niordson (Ref. 5), was to find the optimal beam shape that will maximize the fundamental vibration frequency of a cantilever. Upon reexamination of the problem, it has been found that the original analysis and solution procedure can be simplified and improved substantially. Specifically, the time-consuming inner loop devised for solving the Lagrange multiplier in the original work has been proved to be totally unnecessary and thus should not be considered in the problem solution. This conclusion has led to a new set of simplified equations for the construction of iteration schemes. New asymptotic expressions for the optimum design solution have been obtained and verified by numerical results. Numerical analysis has shown a significant improvement in convergence rate by the proposed new procedure. Also some obvious numerical errors in the original paper have been identified and corrected.

Key Words. Cantilever beams, flexible manipulators, optimum design, fundamental frequency, successive iterations.

1. Introduction

Since the pioneering works by Beesack (Ref. 1), Schwarz (Refs. 2–3), and especially Niordson (Ref. 4), considerable progress has been made in
the optimum design of vibrating elastic structures. Niordson first showed in Ref. 4 that, for simply supported beams with geometrically similar cross sections, an increase of 6.6% in the lowest frequency of vibration can be achieved through optimum shape design. Later, Karihaloo and Niordson (Ref. 5) studied the optimum design of vibrating cantilever beams and found considerably larger increases in the lowest frequency. For example, the lowest frequency of the optimum cantilever with geometrically similar cross sections is 578% larger than that of the corresponding one with a uniform cross section. Similar work and extension have also been conducted by many other researchers (Refs. 6–9). Since then, the problem has become a classical one and has been used widely in the literature and textbooks in structural optimization.

Recently, Wang (Ref. 10) has investigated the problem of the optimum shape design of flexible manipulators. The objective is to increase the fundamental vibration frequency of a flexible manipulator so that a larger bandwidth can be obtained for the manipulator control system. The problem formulation is almost the same as that in Ref. 5. However, different boundary conditions have made the optimization problem for flexible manipulators much more difficult than the corresponding one for cantilever beams.

Initially, we attempted to follow the iteration schemes in Ref. 5 in order to solve the corresponding optimization problem for flexible manipulators. However, for all cases tried, the iterative schemes of Ref. 5 did not converge. It was also found that the implicit equation (so-called inner loop) for solving the Lagrange multiplier in those schemes took a significant amount of computation time. After careful reexamination of the original problem, we found that the time-consuming inner loop in the original iteration schemes was redundant and could be removed completely in the solution process. Eliminating this redundant equation from the iteration process leads to a new formulation for the iterations, and consequently to substantial simplification of the iteration equations and significant improvement in convergence rates. For example, three simplified iteration schemes are needed in this paper to solve the optimum design problem completely, whereas five different sophisticated schemes were required in Ref. 5. These results have offered useful information for solving the optimization problem for manipulators (Refs. 10 and 11).

As in Ref. 5, we assume throughout this paper the following relationship between the moment of inertia $I$ and the area $A$ of a cross section of the beam:

$$ I = cA^p(x), \quad p \geq 1, $$

(1)

where $c$ is a constant. Three cases (viz., $p = 1, 2, 3$) are especially interesting to us, since they correspond to beams with rectangular cross sections of