Equilibrium Solutions for Multiobjective Bimatrix Games Incorporating Fuzzy Goals

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Abstract. Equilibrium solutions in terms of the degree of attainment of a fuzzy goal for games in fuzzy and multiobjective environments are examined. We introduce a fuzzy goal for a payoff in order to incorporate ambiguity of human judgments and assume that a player tries to maximize his degree of attainment of the fuzzy goal. A fuzzy goal for a payoff and the equilibrium solution with respect to the degree of attainment of a fuzzy goal are defined. Two basic methods, one by weighting coefficients and the other by a minimum component, are employed to aggregate multiple fuzzy goals. When the membership functions are linear, computational methods for the equilibrium solutions are developed. It is shown that the equilibrium solutions are equal to the optimal solutions of mathematical programming problems in both cases. The relations between the equilibrium solutions for multiobjective bimatrix games incorporating fuzzy goals and the Pareto-optimal equilibrium solutions are considered.

Key Words. Bimatrix games, multiple payoff matrices, equilibrium solutions, Pareto optimality, mathematical programming problems.

1. Introduction

We consider a two-person non zero-sum bimatrix game with single and multiple payoffs. Such game is called a non zero-sum or general-sum game,

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which includes the zero-sum game as a special case, and is also referred to
as a bimatrix game because it can be expressed as a pair of payoff matrices.
Cooperation between the players can be seen in such situations, but in this
paper the noncooperative case will be treated.

For studies on equilibrium solutions of multiobjective games, Wierzbicki (Ref. 1) defined equilibrium solutions based on order relations, using
several preference cones and optimality criteria such as Pareto optimality
for noncooperative multiobjective n-person games with nonlinear payoff
functions. Furthermore, he analyzed theoretically the relations between equi-
librium solutions for multiobjective games and equilibrium solutions for
single-objective proxy games with payoffs equal to scalarizing functions.
Corley (Ref. 2) defined equilibrium solutions for multiobjective bimatrix
games by using $\mathbb{R}^n \setminus \{0\}$ as a preference cone and developed a method for
computing the solutions. Borm, Tijs, and Van Den Aarssen (Ref. 3) defined
a proxy single-objective game with payoffs equal to a scalarizing function
with weighting coefficients in multiobjective bimatrix games and discussed
the existence of equilibrium solutions for the original multiobjective bimatrix
game through the existence of the equilibrium solutions for the single-objective
proxy game. No studies, however, have ever been tried for multiobjective
games in fuzzy environments.

We will examine equilibrium solutions in terms of the degree of attain-
ment of a fuzzy goal for games in fuzzy and multiobjective environments.
First, we introduce a fuzzy goal for a payoff in order to incorporate the
ambiguity of human judgments and assume that a player tries to maximize
the degree of attainment of the fuzzy goal as we did in Ref. 4.

In Section 2, a fuzzy goal for a payoff and the equilibrium solution
with respect to the degree of attainment of the fuzzy goal are defined. In
Section 3, two basic methods, one by weighting coefficients and the other
by a minimum component, are employed to aggregate multiple fuzzy goals.
When the membership functions are linear, computational methods for the
equilibrium solutions are developed. It is shown that the equilibrium solu-
tions are equal to the optimal solutions of mathematical programming prob-
lems in both cases. This means that we can obtain the equilibrium solutions
by solving mathematical programming problems. In Section 4, we consider
the relation between the equilibrium solutions for multiobjective bimatrix
games incorporating fuzzy goals and the Pareto-optimal equilibrium solu-
tions defined in Borm, Tijs, and Van Den Aarssen (Ref. 3) or Wierzbicki
(Ref. 1). The set of Pareto-optimal equilibrium solutions in such games often
contains sets of continuum power; we can, however, select restricted and
reasonable solutions on the assumption that a player has fuzzy goals and
tries to maximize the degree of attainment for the fuzzy goals.