Equilibrium Solutions for Multiobjective Bimatrix Games Incorporating Fuzzy Goals

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Abstract. Equilibrium solutions in terms of the degree of attainment of a fuzzy goal for games in fuzzy and multiobjective environments are examined. We introduce a fuzzy goal for a payoff in order to incorporate ambiguity of human judgments and assume that a player tries to maximize his degree of attainment of the fuzzy goal. A fuzzy goal for a payoff and the equilibrium solution with respect to the degree of attainment of a fuzzy goal are defined. Two basic methods, one by weighting coefficients and the other by a minimum component, are employed to aggregate multiple fuzzy goals. When the membership functions are linear, computational methods for the equilibrium solutions are developed. It is shown that the equilibrium solutions are equal to the optimal solutions of mathematical programming problems in both cases. The relations between the equilibrium solutions for multiobjective bimatrix games incorporating fuzzy goals and the Pareto-optimal equilibrium solutions are considered.

Key Words. Bimatrix games, multiple payoff matrices, equilibrium solutions, Pareto optimality, mathematical programming problems.

1. Introduction

We consider a two-person non zero-sum bimatrix game with single and multiple payoffs. Such game is called a non zero-sum or general-sum game,

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which includes the zero-sum game as a special case, and is also referred to as a bimatrix game because it can be expressed as a pair of payoff matrices. Cooperation between the players can be seen in such situations, but in this paper the noncooperative case will be treated.

For studies on equilibrium solutions of multiobjective games, Wierzbicki (Ref. 1) defined equilibrium solutions based on order relations, using several preference cones and optimality criteria such as Pareto optimality for noncooperative multiobjective \( n \)-person games with nonlinear payoff functions. Furthermore, he analyzed theoretically the relations between equilibrium solutions for multiobjective games and equilibrium solutions for single-objective proxy games with payoffs equal to scalarizing functions. Corley (Ref. 2) defined equilibrium solutions for multiobjective bimatrix games by using \( R^2_0 \) as a preference cone and developed a method for computing the solutions. Borm, Tijs, and Van Den Aarssen (Ref. 3) defined a proxy single-objective game with payoffs equal to a scalarizing function with weighting coefficients in multiobjective bimatrix games and discussed the existence of equilibrium solutions for the original multiobjective bimatrix game through the existence of the equilibrium solutions for the single-objective proxy game. No studies, however, have ever been tried for multiobjective games in fuzzy environments.

We will examine equilibrium solutions in terms of the degree of attainment of a fuzzy goal for games in fuzzy and multiobjective environments. First, we introduce a fuzzy goal for a payoff in order to incorporate the ambiguity of human judgments and assume that a player tries to maximize the degree of attainment of the fuzzy goal as we did in Ref. 4.

In Section 2, a fuzzy goal for a payoff and the equilibrium solution with respect to the degree of attainment of the fuzzy goal are defined. In Section 3, two basic methods, one by weighting coefficients and the other by a minimum component, are employed to aggregate multiple fuzzy goals. When the membership functions are linear, computational methods for the equilibrium solutions are developed. It is shown that the equilibrium solutions are equal to the optimal solutions of mathematical programming problems in both cases. This means that we can obtain the equilibrium solutions by solving mathematical programming problems. In Section 4, we consider the relation between the equilibrium solutions for multiobjective bimatrix games incorporating fuzzy goals and the Pareto-optimal equilibrium solutions defined in Borm, Tijs, and Van Den Aarssen (Ref. 3) or Wierzbicki (Ref. 1). The set of Pareto-optimal equilibrium solutions in such games often contains sets of continuum power; we can, however, select restricted and reasonable solutions on the assumption that a player has fuzzy goals and tries to maximize the degree of attainment for the fuzzy goals.