Stochastic Bargaining Models

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Abstract. Two stochastic models of labor-management negotiations are proposed and solved explicitly. The cost function considered is such that either a penalty is incurred as long as negotiations take place, or an infinite penalty is incurred if a settlement is not reached before a fixed time. Furthermore, the cost function takes the risk sensitivity of the optimizers into account.

Key Words. Stochastic optimal control, risk sensitivity, Brownian motion, lognormal distribution.

1. Introduction

In Ref. 1, Leitmann considered the following differential game model of collective bargaining:

\[ \dot{o}(t) = u(t), \quad o(0) = o_0, \]  
\[ \dot{d}(t) = -v(t), \quad d(0) = d_0, \]

where \( o(t) \) denotes the offer by management at time \( t \) of total wages per unit time and \( d(t) \) denotes the demand by labor at time \( t \) for total wages per unit time. Furthermore, \( o_0 \) and \( d_0 \) are the initial offer and demand, respectively. The game ends the first time \( d(t) - o(t) = 0 \). The variables \( u(t) \) and \( v(t) \) are the concessions rates, controlled by the two parties. Moreover, there is a variable \( w(t) \) equal to 1, if there is a strike at time \( t \), and equal
to 0 otherwise. The aim, in the case of management, is to choose \( u(t) \) so as to minimize the final offer \( o(T) \) and the potential profit loss during a strike,

\[
\int_0^T w(t)[k - d(t)] \, dt, \tag{3}
\]

and to maximize the potential wage loss during a strike,

\[
\int_0^T w(t)o(t) \, dt, \tag{4}
\]

whereas labor wants to maximize \( d(T) \) and the potential profit loss during a strike, and to minimize the potential wage loss during a strike. In (3), \( k \) is the gross profit (i.e., the profit before the payment of wages) of the company per unit time, which is assumed to be constant, and

\[
T \left( = T(o_0, d_0) \right) = \inf\{t \geq 0: d(t) - o(t) = 0 \mid o(0) = o_0, d(0) = d_0\}. \tag{5}
\]

Leitmann obtained, under some conditions, the equilibrium strategies, which depend on the sign of \( k - 2 \), and showed that there will be a strike [respectively, no strike] if

\[
o^*(t) < k - d^*(t) \quad [\text{resp. } o^*(t) > k - d^*(t)], \tag{6}
\]

where \( o^*(t) \) and \( d^*(t) \) are the optimal strategies. In the case where \( o^*(t) = k - d^*(t) \), there may but need not be a strike.

In this paper, we denote by \( x(t) \) [resp. \( y(t) \)] the percentage of pay increase demanded by labor [resp., offered by management] at time \( t \), and we consider the following model:

\[
dx(t) = -u(t) \, dt + dW_1(t), \tag{7}
\]

\[
dy(t) = v(t) \, dt + dW_2(t), \tag{8}
\]

where \( u(t) \) and \( v(t) \) are the concession rates controlled by labor and management, respectively, and \( W_1(t) \) and \( W_2(t) \) are independent Wiener processes or Brownian motion processes with zero mean and infinitesimal variance \( \epsilon \geq 0 \).

Let

\[x(0) = x \geq y(0) = y.\]

We define

\[
T \left( = T(x, y) \right) = \inf\{t \geq 0: x(t) = y(t) \mid x(0) = x, y(0) = y\}. \tag{9}
\]

Our aim is to find the values \( u^*(t) \), \( v^*(t) \) of \( u(t) \), \( v(t) \) that minimize the cost criterion

\[
C(\theta) \left( = C(x, y, \theta) \right) = -\left(1/\theta\right) \log(E\{\exp[-\theta J(x, y)]\}), \tag{10}
\]